UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

NEWTONS BACKWARD DIFFERENCE INTERPOLATION

The population of a town is as follows

Year	1941	1951	1961	1971	1981	1991
Population (in lakhs)	20	24	29	36	46	51

Estimate the population increase during the period 1946 and 1976.

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
1941	20					
		4				
1951	24		1			
		5		1		
1961	29		2		0	
		7		1		-9
1971	36		3		-9	
		10		-8		
1981	46		-5			
		5				
1991	51					

$$u = \frac{x - x_0}{h} = \frac{1946-1941}{10} = \frac{5}{10}$$

$$u = 0.5$$

Since $x = 1946$, we use Newton's forward Interpolation formula

$$f(x) = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \frac{u(u-1)(u-2)(u-3)}{4!}\Delta^4 y_0 + \frac{u(u-1)(u-2)(u-3)(u-4)}{5!}\Delta^5 y_0$$

$$= 20 + \frac{0.5}{1!}(4) + \frac{(0.5)(0.5-1)}{2!}(1) + \frac{(0.5)(0.5-1)(0.5-2)}{3!}(1) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)}{4!}(0) + \frac{(0.5)(0.5-1)(0.5-2)(0.5-3)(0.5-4)}{5!}(-9)$$

$$= 20 + 2 - 0.125 + 0.0625 + 0 - 0.2460$$

$$f(x) = 21.6915$$

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ii) Since $x = 1976$, we use backward interpolation formula.

$$f(y) = y_n + \frac{u}{1!}\nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n + \cdots$$

$$u = \frac{x - x_n}{h} = \frac{1976 - 1991}{10} = \frac{-15}{10} = -1.5$$

$$f(x) = 51 + \frac{(-1.5)}{1!}(5) + \frac{(-1.5)(-1.5+1)}{2!}(-5) + \frac{(-1.5)(-1.5+1)(-1.5+2)}{3!}$$

$$+ \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)}{4!}(-9) + \frac{(-1.5)(-1.5+1)(-1.5+2)(-1.5+3)(-1.5+4)}{5!}$$

$$= 51 - 7.5 + \frac{(-3.75)}{2} - \frac{3}{6} - \frac{5.0625}{24} - \frac{12.6562}{120}$$

$$= 51 - 7.5 - 1.875 - 0.5 - 0.2109 - 0.10546$$

$$= 40.809$$

Find the value of y at $x = 21$ and $x = 28$ from the following data

x	20	23	26	29
y	0.3420	0.3907	0.4384	0.4848

$h = 3$

$$u = \frac{x - x_0}{h} = \frac{21 - 20}{3} = 1/3.$$

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Difference table

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$
20	0.3420			
		0.4087		
23	0.3907		−0.001	
		0.0477		0.0004
26	0.4384		−0.013	
		0.0464		
29	0.4848			

$$y = y_0 + \frac{u}{1!}\Delta y_0 + \frac{u(u-1)}{2!}\Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!}\Delta^3 y_0 + \dots$$

$$= 0.3420 + \left(\frac{0.33}{1!}\right)(0.0487) + \frac{(0.33)(0.33-1)}{2!}(-0.001)$$

$$+ \frac{(0.33)(0.33-1)(0.33-2)}{3!}(0.0004)$$

$$= 0.3420 + 0.01607 + 0.00011 + 0.00018$$

$$= 0.358.$$

Since $x = 28$ is close to x_n we use Newton's Backward Interpolation formula.

$$f(y) = y_n + \nabla y_n + \frac{u(u+1)}{2!}\nabla^2 y_n + \frac{u(u+1)(u+2)}{3!}\nabla^3 y_n$$

$$u = \frac{x - x_n}{h} = \frac{28-29}{3} = -0.33.$$

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$$f(x) = 0.4848 + \frac{(-0.33)(0.0464)}{1!} + \frac{(-0.33)(-0.33+1)}{2}(-0.001) + \frac{(-0.33)(-0.33+1)(-0.33+2)}{6}(-0.0003)$$

$$= 0.4848 - 0.015312 + 0.0001437 + 0.000001846$$

$$f(x) = 0.469$$