UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

APPROXIMATION OF DERIVATIVES USING INTERPOLATION POLYNOMIALS

Numerical Differentiation :-

It is a process of computing the value of the derivative $\frac{dy}{dx}$ for some particular value of x, from the given data (x_i, y_i). If the values of x are equally spaced, we can use Newton's Interpolation formula for equal intervals. If the values of x are unequally spaced we can use Lagrange's Interpolation formula (or) Newton's divided difference interpolation formula.

Differentiation using interpolation formula:

Newton's forward difference formula to compute the derivatives

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{\Delta^2 y_0}{3} - \frac{\Delta^4 y_0}{4} + \dots\right]$$

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$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \left(\frac{d^2y}{dx^2}\right)_{u=0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12} y_0 + \cdots\right]$$

$$\frac{d^3y}{dx^3} = \left(\frac{d^3y}{dx^3}\right)_{u=0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots\right]$$

When $u > 0$

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{6}\Delta^2 y_0 + \frac{(4u^3-18u^2+22u-6)}{24}\Delta^4 y_0\right.$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 + \frac{(6u^2-18u+11)}{12}\Delta^4 y_0 + \cdots\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\Delta^3 y_0 + \frac{12u-18}{12}\Delta^4 y_0 + \cdots\right], \text{ where } u = \frac{x-x_0}{h}.$$

Newton's backward difference formula to compute the derivatives:

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \left(\frac{dy}{dx}\right)_{u=0} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4} + \cdots\right]$$

When u (or) $v > 0$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{(2v+1)}{2}\nabla^2 y_n + \frac{(3v^2+6v+2)}{6}\nabla^3 y_n + \frac{(4v^3+18v^2+22v+6)}{24}\nabla^4 y_n\right]$$

where $v = \frac{x-x_n}{h}$.