UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

APPROXIMATION OF DERIVATIVES USING INTERPOLATION POLYNOMIALS

The population of a certain town is given below. Find the rate of growth of population in 1931, 1941, 1961 and 1971.

Year (x):	1931	1941	1951	1961	1971
population in thousands (y):	40.62	60.82	79.95	103.56	132.65

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$
1931	40.62				
		20.18			
1941	60.82		−1.03		
		19.15		5.49	
1951	79.95		4.46		−4.47
		23.61		1.02	
1961	103.56		5.48		
		29.09			
1971	132.65				

To find $f'(1931)$ and $f'(1941)$ we use Newton's forward derivative formula:

$$f'(1931) = \left(\frac{dy}{dx}\right)_{x=0} = \frac{1}{h}\left\{\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{\Delta^3 y_0}{3} - \frac{\Delta^4 y_0}{4}\right\}$$

here $h = 10$

$$= \frac{1}{10}\left\{(20.18) - \frac{1}{2}(-1.03) + \frac{1}{3}(5.49) - \frac{1}{4}(-4.47)\right\}$$

$$= \frac{1}{10}\{20.18 + 0.515 + 1.83 + 1.1175\}$$

$$f'(1931) = 2.36425$$

UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

APPROXIMATION OF DERIVATIVES USING INTERPOLATION POLYNOMIALS

$f'(1941) \Rightarrow u = \frac{x - x_0}{h} = \frac{1941-1931}{10} = 1.$

$$\left(\frac{dy}{dx}\right)_{u=1} = \frac{1}{h}\left\{\Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \frac{(3u^2-6u+2)}{6}\Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{24}\Delta^4 y_0\right\}$$

$$= \frac{1}{10}\left\{(20.18) + \frac{(2-1)}{2}(-1.03) + \frac{3-6+2}{6}(5.49) + \frac{4-18+22-6}{24}(-4.47)\right\}$$

$$= \frac{1}{10}\left\{20.18 - 0.515 - 0.915 - 0.3725\right\}$$

$$= \frac{1}{10}\left\{18.3775\right\}$$

$f'(1941) = 1.83775$

To find $f'(1961)$ and $f'(1971)$ we use Newtons backward derivative formula.

$f'(1961)$: Here $v = \frac{x - x_n}{h} = \frac{1961-1971}{10} = -1$

$$\frac{dy}{dx} = \frac{1}{h}\left\{\nabla y_n + \frac{2v+1}{2}\nabla^2 y_n + \frac{3v^2+6v+2}{6}\nabla^3 y_n + \frac{4v^3+18v^2+22v+6}{24}\nabla^4 y_n\right\}$$

$$= \frac{1}{10}\left\{(29.09) + \frac{2(-1)+1}{2}(5.48) + \frac{3(-1)^2+6(-1)+2}{6}(1.02) + \frac{4(-1)^3+18(-1)^2+22(-1)+6}{24}(-4.47)\right\}$$

UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

APPROXIMATION OF DERIVATIVES USING INTERPOLATION POLYNOMIALS

$$= \frac{1}{10}\left\{29.09 - \frac{1}{2}(5.48) + \frac{3(1)-6+2}{6}(1.02) + \frac{4(-1)+18+22+6}{24}(-4.4\ldots\right\}$$

$$= \frac{1}{10}\{29.09 - 2.74 - 0.17 + 0.3725\}$$

$$= \frac{1}{10}\{26.5525\}$$

$$f'(1961) = 2.65525$$

$f'(1971) \Rightarrow$ Here $v = \dfrac{1971-1971}{10} = 0.$

$$\frac{dy}{dx} = \frac{1}{h}\left\{\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{\nabla^3 y_n}{3} + \frac{\nabla^4 y_n}{4}\right\}$$

$$= \frac{1}{10}\left\{29.09 + \frac{1}{2}(5.48) + \frac{1}{3}(1.02) + \frac{1}{4}(-4.47)\right\}$$

$$= \frac{1}{10}\{29.09 + 2.74 + 0.34 - 1.1175\}$$

$$= \frac{1}{10}\{31.0525\}$$

$$f'(1971) = 3.10525.$$

2. A jet fighters position on an aircraft carriers runway was timed during landing

t(sec) :	1.0	1.1	1.2	1.3	1.4	1.5	1.6
y(m) :	7.989	8.403	8.781	9.129	9.451	9.750	10.031

where y is the distance from the end of the carrier. Estimate velocity $\left(\frac{dy}{dx}\right)$ and acceleration $\left(\frac{d^2y}{dx^2}\right)$ at i) $t=1.1$ (ii) $t=1.6$ using numerical differentiation

UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION
APPROXIMATION OF DERIVATIVES USING INTERPOLATION POLYNOMIALS

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$	$\Delta^6 y$
1.0	7.989						
		0.414					
1.1	8.403		-0.036				
		0.378		0.006			
1.2	8.781		-0.03		-0.002		
		0.348		0.004		0.001	
1.3	9.129		0.026		-0.001		0.002
		0.322		0.003		0.003	
1.4	9.451		-0.023		-0.002		
		0.299		0.005			
1.5	9.750		-0.018				
		0.281					
1.6	10.031						

To find $t = 1.1 \quad h = 0.1$

$$u = \frac{1.1 - 1.0}{0.1} = 1$$

$$\left(\frac{dy}{dx}\right)_{t=1.1} = \left(\frac{dy}{dx}\right)_{u=1} = \frac{1}{h}\left[\Delta y_0 + \frac{(2u-1)}{2}\Delta^2 y_0 + \left(\frac{3u^2 - 6u + 2}{6}\right)\Delta^3 y_0 + \left(\frac{4u^3 - 18u^2 + 22u - 6}{24}\right)\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{0.1}\left[0.414 + \left(\frac{2-1}{2}\right)(-0.036) + \left(\frac{3-6+2}{6}\right)(0.0006) + \left(\frac{4-18+22-6}{24}\right)(-0.002) + \cdots\right]$$

$$= 3.9483$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + (u-1)\Delta^3 y_0 \left(\frac{6u^2 - 18u + 11}{12}\right)\Delta^4 y_0 + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.036 + (1-1)(0.006) + \left(\frac{6-18+11}{12}\right)(-0.002)\right]$$

$$= -3.5833$$

UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

APPROXIMATION OF DERIVATIVES USING INTERPOLATION POLYNOMIALS

To find $t = 1.6$; $v = \frac{t - t_n}{n}$ $\therefore v = 0$

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\right]$$

$$= \frac{1}{0.1}\left[0.281 + \frac{1}{2}(-0.018) + \frac{1}{3}(0.005) + \frac{1}{4}(0.002) + \frac{1}{5}(0.003) + \frac{1}{6}(0.002)\right]$$

$$= 2.751$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$= \frac{1}{(0.1)^2}\left[-0.018 + 0.005 + \frac{11}{12}(0.002)\right]$$

$$= -1.1167$$