UNIT 4 INTERPOLATION,NUMERICAL DIFFERENTIATION AND NUMERICAL INTEGRATION

NUMERICAL SINGLE INTERGRATION USING TRAPEZOIDAL RULE

Numerical Integration by Trapezoidal Rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[(y_0+y_n) + 2(y_1+y_2+\dots+y_{n-1})\right]$$

$$= \frac{h}{2}\left[A+2B\right]$$

Where A = Sum of the first & last ordinates

B = Sum of the Remaining Ordinates

$$h = \frac{b-a}{n}$$

Problems:

1. Using trapezoidal rule evaluate $\int_{-1}^{1} \frac{dx}{1+x^2}$ taking 8 intervals.

$$y(x) = \frac{1}{1+x^2} \qquad h = \frac{b-a}{n} = \frac{1+1}{8} = \frac{2}{8} = 0.25$$

x	-1	-0.75	-0.5	0.25	0	0.25	0.5	0.75	1
y	0.5	0.64	0.8	0.94	1	0.94	0.8	0.64	0.5

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By trapezoidal Rule:

$$\int_{x_0}^{x_n} y\,dx = \frac{h}{2}\left[A + 2B\right]$$

$$\int_{-1}^{1} \frac{dx}{1+x^2} = \frac{0.25}{2}\Big[(0.5+0.5) + 2(0.64 + 0.8 + 0.94 + 1 + 0.94 + 0.8 + 0.64)\Big]$$

$$= \frac{0.25}{2}\left[1 + 2(5.76)\right]$$

$$= \frac{0.25}{2}(12.52)$$

$$= 1.565$$

2. Evaluate $\int_0^1 \frac{dx}{1+x^2}$ with $h = \frac{1}{6}$ by trapezoidal Rule

$y = \frac{1}{1+x^2}$ $\quad h = \frac{b-a}{n}$ $\quad \Rightarrow \frac{1}{6} = \frac{1-0}{n}$ $\quad \Rightarrow n = 6$

x	0	$1/6$	$2/6$	$3/6$	$4/6$	$5/6$	1
y	1	$\frac{36}{37}$	$\frac{9}{10}$	$\frac{4}{5}$	$\frac{9}{13}$	$\frac{36}{61}$	$\frac{1}{2}$

By Trapezoidal Rule:

$$\int_0^1 \frac{dx}{1+x^2} = \frac{1/6}{2}\left[\left(1+\frac{1}{2}\right) + 2\left(\frac{36}{37} + \frac{9}{10} + \frac{4}{5} + \frac{9}{13} + \frac{36}{61}\right)\right]$$

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NUMERICAL SINGLE INTERGRATION USING TRAPEZOIDAL RULE

$$= \frac{1}{12}\left[1.5 + 2(0.97 + 0.90 + 0.80 + 0.69 + 0.59)\right]$$

$$= \frac{1}{12}\left(1.5 + 2(3.95)\right)$$

$$= \frac{1}{12}[9.40] = 0.7842$$

3. Dividing the range into 10 equal parts find the value of $\int_0^{\pi/2} \sin x\, dx$ by trapezoidal rule.

$y(x) = \sin x$, $h = \frac{b-a}{n} = \frac{\pi/2 - 0}{10} = \frac{\pi}{20}$

x	0	$\frac{\pi}{20}$	$\frac{2\pi}{20}$	$\frac{3\pi}{20}$	$\frac{4\pi}{20}$	$\frac{5\pi}{20}$	$\frac{6\pi}{20}$	$\frac{7\pi}{20}$	$\frac{8\pi}{20}$	$\frac{9\pi}{20}$	$\frac{10\pi}{20}$
y	0	0.156	0.309	0.453	0.587	0.707	0.809	0.891	0.951	0.987	1

By trapezoidal rule,

$$\int_0^{\pi/2} \sin x\, dx = \frac{\pi/20}{2}\left[(0+1) + 2(0.156 + 0.309 + 0.453 + 0.587 + 0.707 + 0.809 + 0.891 + 0.951 + 0.987)\right]$$

$$= \frac{\pi}{40}\left[1 + 2(5.85)\right] = \frac{\pi}{40}[1 + 11.7]$$

$$= \frac{\pi}{40}[12.7]$$

$$= \frac{3.14 \times 12.7}{40} = 0.9974$$