## CHI - SQUARE TEST :

$$\chi^2 = \frac{\Sigma [O_i - E_i]^2}{E_i}$$

where $O_i \rightarrow$ Observed frequency

$E_i \rightarrow$ Experimental frequency or Expected frequency $= \frac{\Sigma O_i}{n}$

degrees of freedom, $\nu = n - 1$

properties :

i) The mean of $\chi^2$ dist. is equal to the no. of degrees of freedom
ii) The variance of $\chi^2$ dist. is twice the degrees of freedom
iii) If $\chi^2$ is a chi-square variate with $\nu$ degrees of freedom, then $\chi^2/2$ is a gamma variate with parameter $\nu/2$.
iv) standard $\chi^2$ variate tends to standard normal variate as $n \rightarrow \infty$.

Applications :

i) To test if the hypothetical value of the population variance is $\sigma^2 = \sigma_0^2$
ii) To Test the goodness of fit.
iii) To test the Independence of attributes.
iv) To Test the homogeneity of indep. estimates of the population variance.

Degrees of freedom : no. of values in a set which may be assigned arbitrarily.

1) The table below gives the number of aircraft accidents that occurred during the various days of the week. Test whether the accidents are uniformly distributed over the week.

| Days | Mon | Tues | Wed | Thurs | Fri | Sat |
|---|---|---|---|---|---|---|
| No. of accidents | 14 | 18 | 12 | 11 | 15 | 14 |

Soln:

given, total no of accidents = 84

No. of days = 6

$\therefore$ Expected frequencies of the accidents $= E_i = \frac{84}{6}$

$= 14$

| $O_i$ | $E_i$ | $(O_i - E_i)^2$ | $\frac{(O_i - E_i)^2}{E_i}$ |
|---|---|---|---|
| 14 | 14 | 0 | $0/14 : 0$ |
| 18 | 14 | 16 | $16/14 : 1.14$ |
| 12 | 14 | 4 | $4/14 : 0.285$ |
| 11 | 14 | 9 | $9/14 : 0.642$ |
| 15 | 14 | 1 | $1/14 : 0.071$ |
| 14 | 14 | 0 | $0/14 : 0$ |

$$\sum \frac{(O_i - E_i)^2}{E_i} = 2.14285$$

Step 1: Formulate $H_0$ & $H_1$:

$H_0$: The accidents are uniformly distributed.

$H_1$: The accidents are not uniformly distributed.

Step 2: Los at $\alpha = 5\%$.

Step 3: Test statistic, $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i} = 2.1428$

Step 4: Degrees of freedom, $\nu = n-1$

$= 6-1$

$= 5$

Tab value is $11.07 = \chi^2_\alpha$

Step 5: Conclusion:

$\chi^2 = 2.1428 < 11.07 = \chi^2_\alpha$

$\therefore$ $H_0$ is accepted at 5% los. (ie) the accident are uniformly distributed.

2) A die was thrown 498 times. Denoting $x$ to be the number appearing on the top face of it, the observed frequency of $x$ is given below:

| $x$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $f$ | 69 | 78 | 85 | 82 | 86 | 98 |

what opinion you would form for the accuracy of the die?

Soln:

Given, Expected frequency, $E_i = \frac{\text{Total frequency}}{6}$

$= \frac{498}{6} = 83$

| Ex | $O_i$ | $E_i$ | $(O_i-E_i)^2$ | $(O_i-E_i)^2/E_i$ |
|---|---|---|---|---|
| 1 | 69 | 83 | 196 | 2.3614 |
| 2 | 78 | 83 | 25 | 0.3012 |
| 3 | 85 | 83 | 4 | 0.0481 |
| 4 | 82 | 83 | 1 | 0.0120 |
| 5 | 86 | 83 | 9 | 0.1084 |
| 6 | 98 | 83 | 225 | 2.7108 |

$$\frac{\Sigma (O_i-E_i)^2}{E_i} = 5.5419$$

step 1: Formulate $H_0$ & $H_1$:

$H_0$: A die is unbiased

$H_1$: A die is not unbiased (ie) biased.

step 2: LOS at $\alpha = 5\%$.

step 3: Test statistic, $\chi^2 = \frac{\Sigma (O_i-E_i)^2}{E_i} = 5.542$.

step 4: Degrees of freedom, $\nu = n-1$
$= 6-1$
$= 5$

$\therefore \chi^2_\alpha = 11.04$.

step 5: Conclusion;

$\chi^2 = 5.542 < 11.04 = \chi^2_\alpha$

$\therefore$ $H_0$ is accepted at 5% LOS (ie) A die is unbiased