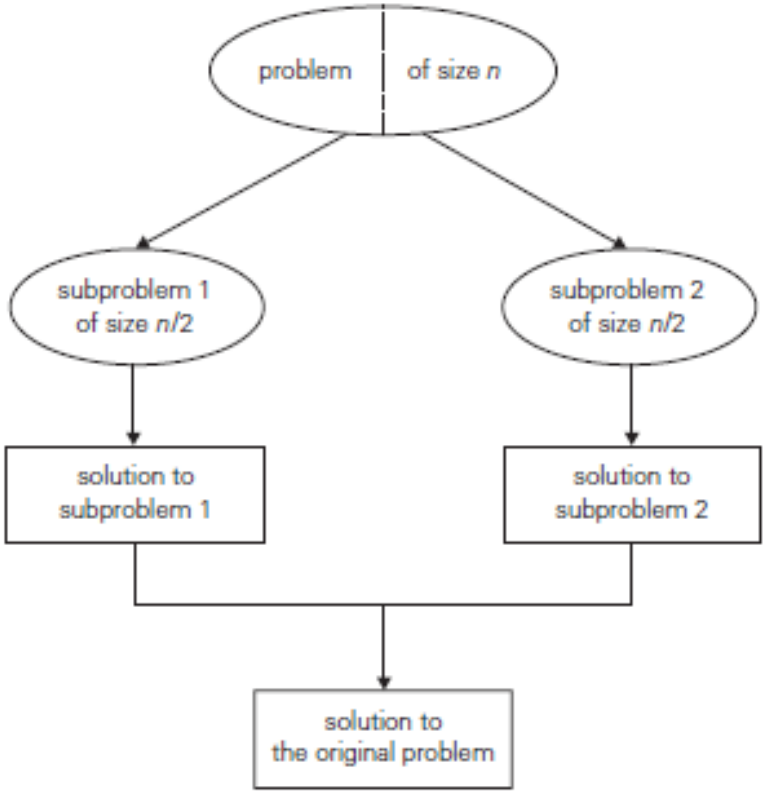
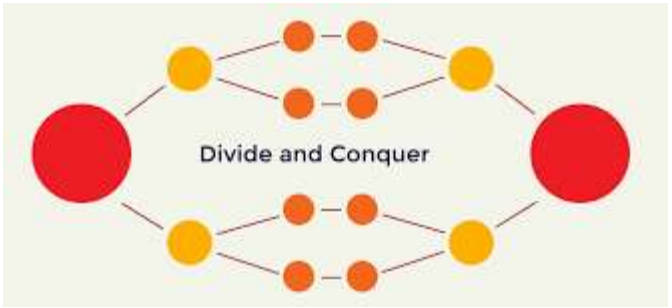
Unit II – Divide and Conquer

- **Merge sort**
- Quick sort
- Binary search
- Multiplication of large Integers
- Strassen's Matrix Multiplication

Divide and Conquer Design Technique





Divide and Conquer Design Technique

1. A problem is divided into several **sub problems** of the same type, ideally of about equal size.

2. The sub problems are **solved** (typically ***recursively***, though sometimes a different algorithm is employed, especially when sub problems become small enough).

3. If necessary, the solutions to the **sub problems are combined** to get a solution to the original problem.

Algorithm for Divide and Conquer:

```
DAC(P)
if small(P)
    S(P)
else
    Divide P into P1,P2…..Pn
        Apply DAC(P1), DAC(P2)…..DAC(Pn)
        S(DAC(P1), DAC(P2)…..DAC(Pn))
```

Divide and Conquer Design Technique

Recurrence Relation

$T(n) = a\,T(n/b) + f(n)$

here

T(n/b) – sub problem

f(*n*) – time spent for dividing n into n/b and combing their solutions

Masters Theorem

$T(n) = a\,T(n/b) + f(n)$ $a>=1, b>1, f(n) = O(n^k \log^p n)$

Find values: 1. $\log_b a$

2. k

Case 1 : if $\log_b a > k$, then $O(n^{\log_b a})$

Case 2: if $\log_b a = k$, then $O(n^k \log^p n \log n)$

Case 3: if $\log_b a < k$, then $O(n^k \log^p n)$

- ***Masters Theorem - Example***
- $T(n) = a\,T(n/b) + f(n)$, $f(n) = O(n^k \log^p n)$
- $T(n) = 2\,T(n/2) + 1$
- Here $a = 2$, $b = 2$, $f(n) = 1 = O(1) = O(n^0 \log^0 n)$
- From this $k = 0$, $p = 0$, $a=2$, $b=2$

Find values: 1. $\log_b a = \log_2 2 = 1$

2. $k = 0$

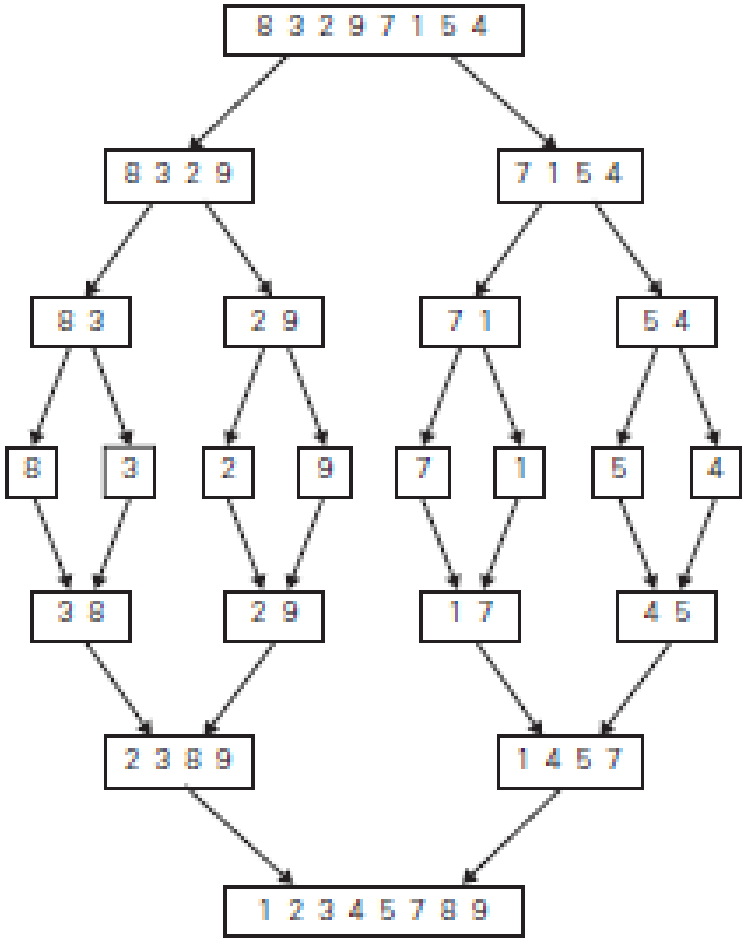
Case 1: $\log_b a > k \rightarrow 1 > 0$

$O(n^{\log_b a})$

$O(n^1)$

MERGE SORT - Example

- Link
- Example



MERGE SORT - Algorithm

ALGORITHM *Mergesort*$(A[0..n-1])$ → T (n)

```
//Sorts array A[0..n − 1] by recursive mergesort
//Input: An array A[0..n − 1] of orderable elements
//Output: Array A[0..n − 1] sorted in nondecreasing order
if n > 1
    copy A[0..⌊n/2⌋ − 1] to B[0..⌊n/2⌋ − 1]
    copy A[⌊n/2⌋..n − 1] to C[0..⌈n/2⌉ − 1]
    Mergesort(B[0..⌊n/2⌋ − 1])        → T (n/2)
    Mergesort(C[0..⌈n/2⌉ − 1])        → T (n/2)
    Merge(B, C, A)   //see below      → n
```

ALGORITHM *Merge*$(B[0..p-1], C[0..q-1], A[0..p+q-1])$

```
//Merges two sorted arrays into one sorted array
//Input: Arrays B[0..p − 1] and C[0..q − 1] both sorted
//Output: Sorted array A[0..p + q − 1] of the elements of B and C
i ← 0; j ← 0; k ← 0
while i < p and j < q do
    if B[i] ≤ C[j]
        A[k] ← B[i]; i ← i + 1
    else A[k] ← C[j]; j ← j + 1
    k ← k + 1
if i = p
    copy C[j..q − 1] to A[k..p + q − 1]
else copy B[i..p − 1] to A[k..p + q − 1]
```

MERGE SORT - Analysis

- $T(n) = 1 \quad n=1$
- $= 2\,T(n/2) + n \quad n > 1$
- Here a=b=2, f(n) = n
- 2 values
  - $Log_b a = log_2 2 = 1$
  - K → $n^k = n^1$
- $log_b a = k$ → 1 = 1→ case 2 → O(n log n)