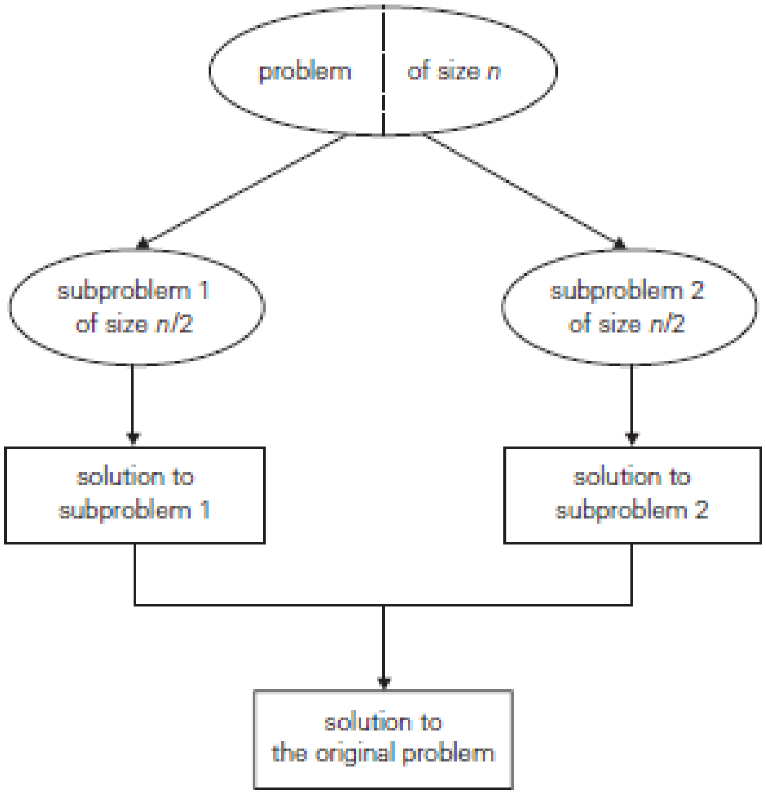
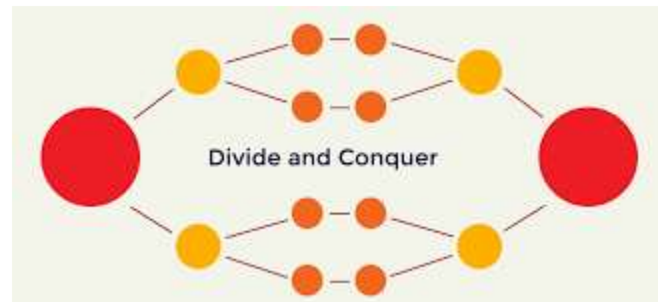
Unit II – Divide and Conquer

- **Merge sort**
- Quick sort
- Binary search
- Multiplication of large Integers
- Strassen's Matrix Multiplication

Divide and Conquer Design Technique





Divide and Conquer Design Technique

1. A problem is divided into several **sub problems** of the same type, ideally of about equal size.

2. The sub problems are **solved** (typically ***recursively***, though sometimes a different algorithm is employed, especially when sub problems become small enough).

3. If necessary, the solutions to the **sub problems are combined** to get a solution to the original problem.

Algorithm for Divide and Conquer:

```
  DAC(P)
if small(P)
    S(P)
else
    Divide P into P1,P2…..Pn
       Apply DAC(P1), DAC(P2)…..DAC(Pn)
       S(DAC(P1), DAC(P2)…..DAC(Pn))
```

Divide and Conquer Design Technique

Recurrence Relation

$T(n) = a\,T(n/b) + f(n)$

here

$T(n/b)$ – sub problem

$f(n)$ – time spent for dividing n into n/b and combing their solutions

Masters Theorem

$T(n) = a\,T(n/b) + f(n)$ $a>=1, b>1, f(n) = O(n^k \log^p n)$

Find values: 1. $\log_b a$

2. k

Case 1 : if $\log_b a > k$, then $O(n^{\log_b a})$

Case 2: if $\log_b a = k$, then $O(n^k \log^p n \log n)$

Case 3: if $\log_b a < k$, then $O(n^k \log^p n)$

- ***Masters Theorem - Example***
- $T(n) = a\,T(n/b) + f(n)$, $f(n) = O(n^k \log^p n)$
- $T(n) = 2\,T(n/2) + 1$
- Here $a = 2$, $b = 2$, $f(n) = 1 = O(1) = O(n^0 \log^0 n)$
- From this $k = 0$, $p = 0$, $a=2$, $b=2$

Find values: 1. $\log_b a = \log_2 2 = 1$

2. $k = 0$

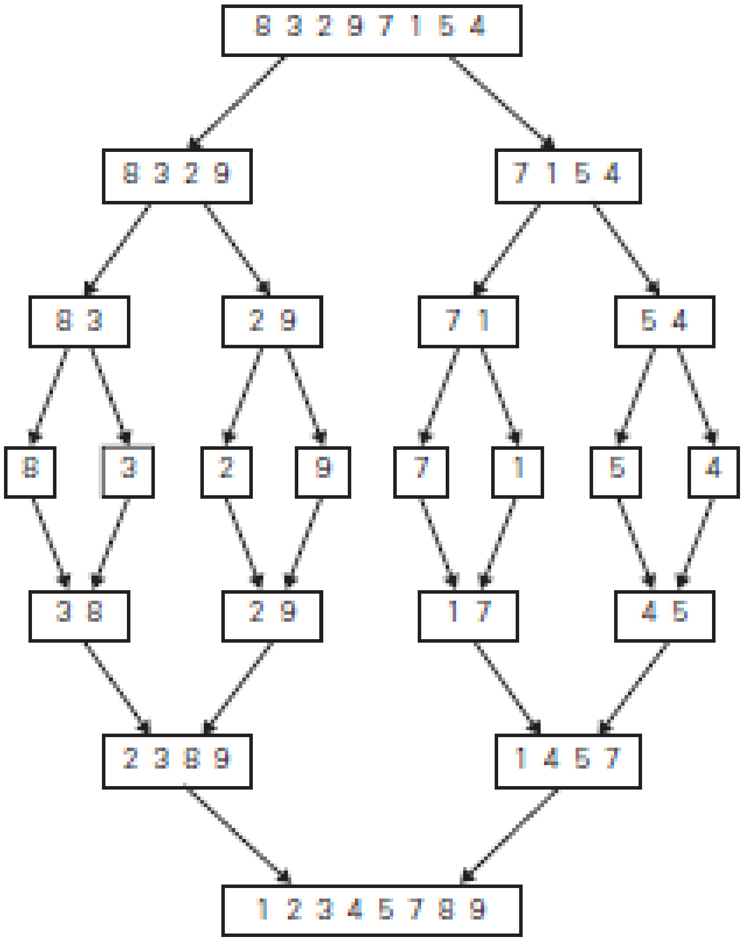
Case 1: $\log_b a > k \rightarrow 1 > 0$

$O(n^{\log_b a})$

$O(n^1)$

MERGE SORT - Example

- Link
- Example



MERGE SORT - Algorithm

ALGORITHM *Mergesort*$(A[0..n-1])$ → T (n)

//Sorts array $A[0..n-1]$ by recursive mergesort
//Input: An array $A[0..n-1]$ of orderable elements
//Output: Array $A[0..n-1]$ sorted in nondecreasing order
if $n > 1$
    copy $A[0..\lfloor n/2 \rfloor - 1]$ to $B[0..\lfloor n/2 \rfloor - 1]$
    copy $A[\lfloor n/2 \rfloor..n-1]$ to $C[0..\lceil n/2 \rceil - 1]$
    Mergesort$(B[0..\lfloor n/2 \rfloor - 1])$ → T (n/2)
    Mergesort$(C[0..\lceil n/2 \rceil - 1])$ → T (n/2)
    Merge(B, C, A) //see below → n

ALGORITHM *Merge*$(B[0..p-1], C[0..q-1], A[0..p+q-1])$

//Merges two sorted arrays into one sorted array
//Input: Arrays $B[0..p-1]$ and $C[0..q-1]$ both sorted
//Output: Sorted array $A[0..p+q-1]$ of the elements of B and C
$i \leftarrow 0$; $j \leftarrow 0$; $k \leftarrow 0$
while $i < p$ **and** $j < q$ **do**
    if $B[i] \le C[j]$
        $A[k] \leftarrow B[i]$; $i \leftarrow i + 1$
    else $A[k] \leftarrow C[j]$; $j \leftarrow j + 1$
    $k \leftarrow k + 1$
if $i = p$
    copy $C[j..q-1]$ to $A[k..p+q-1]$
else copy $B[i..p-1]$ to $A[k..p+q-1]$

MERGE SORT - Analysis

- $T(n) = 1$ $\quad n=1$
- $= 2\,T(n/2) + n$ $\quad n > 1$
- Here a=b=2, f(n) = n
- 2 values
  - $Log_b a = log_2 2 = 1$
  - $K \rightarrow n^k = n^1$
- $log_b a = k \rightarrow 1 = 1 \rightarrow$ case 2 $\rightarrow O(n \log n)$