Unit II – Divide and Conquer

- Merge sort
- Quick sort
- Binary search
- **Multiplication of large Integers**
- **Strassen's Matrix Multiplication**
  - Decrease the no. of multiplications at the expense of a slight increase in no. of additions

Multiplication of Large Integers

Example : 23 14 $n = 2$

$a_0 a_1$ $b_0 b_1$

$n^2 = 4$ Multi

$23 = (2 * 10^1) + (3 \times 10^0)$

$14 = (1 * 10^1) + (4 \times 10^0)$

$23 * 14$

$[(2 * 10^1) + (3 * 10^0)] * [(1 * 10^1) + (4 * 10^0)]$

$(2 * 1) 10^2 + [(2 * 4) + (3 * 1)] 10^1 + (3 * 4) 10^0$.

$8 + 3 = 11$

$(2 * 4) + (3 * 1) = (2 + 3) * (4 + 1) - 2 * 1 - 3 * 4$

a_0 b_1 a_1 b_0

$= 5 * 5 - (2 + 12)$

$= 25 - 14 = 11$

Multiplication of large Integers

$$c = a * b = c_2 10^2 + c_1 10^1 + c_0.$$

$$c_2 = 2 * 1 \Rightarrow a_0 * b_0$$

$$c_0 = 3 * 4 \Rightarrow a_1 * b_1$$

$$c_1 = (a_0 + a_1) * (b_0 + b_1) - (c_2 + c_0)$$

$$c = a * b = c_2 10^{n} + c_1 10^{n/2} + c_0.$$

$M(n) = 3\,M(n/2) \quad \text{for } n > 1$

$M(1) = 1$

$\boxed{n = 2^k} \implies k = \log_2 n$

$M(2^k) = 3\,M(2^k/2^1)$

$= 3M(2^{k-1})$

$= 3[3M(2^{k-2})]$

$= 3^2 M(2^{k-2})$

$= 3^3 M(2^{k-3})$

$= 3^4 M(2^{k-4})$

$= 3^5 M(2^{k-5}) \implies 3^i M(2^{k-i})$

$\implies 3^k M(2^{k-k})$

$\implies 3^k M(2^0)$

$\implies 3^k M(1)$

$\implies 3^k$

$\boxed{a^{\log_b c} = c^{\log_b a}}$

$\implies 3^{\log_2 n}$

$\implies n^{\log_2 3}$

$\implies n^{1.585}$

performed well over integers 600 digits long

Multiplication of large Integers

- *Example;*
- 1234*1234
- 123456*654321

Strassen's Matrix Multiplication

- Strassen - 1969

$$\begin{bmatrix} c_{00} & c_{01} \\ c_{10} & c_{11} \end{bmatrix} = \begin{bmatrix} a_{00} & a_{01} \\ a_{10} & a_{11} \end{bmatrix} * \begin{bmatrix} b_{00} & b_{01} \\ b_{10} & b_{11} \end{bmatrix}$$

$$= \begin{bmatrix} m_1 + m_4 - m_5 + m_7 & m_3 + m_5 \\ m_2 + m_4 & m_1 + m_3 - m_2 + m_6 \end{bmatrix},$$

$$m_1 = (a_{00} + a_{11}) * (b_{00} + b_{11}),$$
$$m_2 = (a_{10} + a_{11}) * b_{00},$$
$$m_3 = a_{00} * (b_{01} - b_{11}),$$
$$m_4 = a_{11} * (b_{10} - b_{00}),$$
$$m_5 = (a_{00} + a_{01}) * b_{11},$$
$$m_6 = (a_{10} - a_{00}) * (b_{00} + b_{01}),$$
$$m_7 = (a_{01} - a_{11}) * (b_{10} + b_{11}).$$

7 Multiplications & 18 additions

Strassen's Matrix Multiplication - Analysis

Since $n = 2^k$,

$$M(2^k) = 7M(2^{k-1}) = 7[7M(2^{k-2})] = 7^2M(2^{k-2}) = \cdots$$
$$= 7^iM(2^{k-i}) \cdots = 7^kM(2^{k-k}) = 7^k.$$

Since $k = \log_2 n$,

$$M(n) = 7^{\log_2 n} = n^{\log_2 7} \approx n^{2.807},$$