In fuzzy logic, operations like intersection (AND) and union (OR) must handle degrees of membership rather than binary values. **T-norms** (triangular norms) and **T-conorms** (triangular conorms) are mathematical functions that generalize these operations while preserving essential properties.

- **T-norm (Triangular Norm)** → Generalized intersection (AND)
- **T-conorm (Triangular Conorm)** → Generalized union (OR)

These functions play a critical role in fuzzy set theory, fuzzy inference, and multi-criteria decision-making.

2. T-Norm (Triangular Norm)

A **T-norm** is a function $T : [0,1] \times [0,1] \to [0,1]$ that represents fuzzy **intersection** (logical AND). It satisfies the following properties:

2.1 Properties of T-Norm

1. **Commutativity**: $T(a,b) = T(b,a)$
2. **Associativity**: $T(a, T(b,c)) = T(T(a,b), c)$
3. **Monotonicity**: If $a \leq c$ and $b \leq d$, then $T(a,b) \leq T(c,d)$
4. **Boundary Condition**: $T(a,1) = a$ and $T(a,0) = 0$

2.2 Common T-Norms

Different T-norms are used based on the required behavior:

T-Norm Type	Definition	Interpretation
Minimum (Gödel T-norm)	$T(a,b) = \min(a,b)$	Most commonly used; standard fuzzy AND
Product T-norm	$T(a,b) = a \times b$	Allows smooth transitions
Łukasiewicz T-norm	$T(a,b) = \max(0, a+b-1)$	Used in fuzzy logic with strong negation
Drastic T-norm	$T(a,b) = \begin{cases} b, & \text{if } a = 1 \\ a, & \text{if } b = 1 \\ 0, & \text{otherwise} \end{cases}$	Extreme case, only 1 survives

2.3 Example

If we take two fuzzy values $A = 0.7$ and $B = 0.5$:

- **Minimum T-norm**: $T(0.7, 0.5) = \min(0.7, 0.5) = 0.5$
- **Product T-norm**: $T(0.7, 0.5) = 0.7 \times 0.5 = 0.35$
- **Łukasiewicz T-norm**: $T(0.7, 0.5) = \max(0, 0.7 + 0.5 - 1) = 0.2$

3. T-Conorm (Triangular Conorm)

A **T-conorm** (also called an **S-norm**) is a function $S : [0,1] \times [0,1] \to [0,1]$ that represents fuzzy **union** (logical OR).

3.1 Properties of T-Conorm

1. **Commutativity**: $S(a,b) = S(b,a)$
2. **Associativity**: $S(a, S(b,c)) = S(S(a,b), c)$
3. **Monotonicity**: If $a \leq c$ and $b \leq d$, then $S(a,b) \leq S(c,d)$
4. **Boundary Condition**: $S(a,0) = a$ and $S(a,1) = 1$

3.2 Common T-Conorms

Different T-conorms provide different interpretations of fuzzy unions:

T-Conorm Type	Definition	Interpretation
Maximum (Gödel S-norm)	$S(a,b) = \max(a,b)$	Standard fuzzy OR
Probabilistic Sum	$S(a,b) = a + b - a \times b$	Used in probability-based fuzzy logic
Łukasiewicz S-norm	$S(a,b) = \min(1, a+b)$	Ensures boundedness
Drastic S-norm	$S(a,b) = \begin{cases} b, & \text{if } a = 0 \\ a, & \text{if } b = 0 \\ 1, & \text{otherwise} \end{cases}$	Extreme case, always 1 unless both are 0

3.3 Example

For $A = 0.7$ and $B = 0.5$:

- **Maximum T-conorm**: $S(0.7, 0.5) = \max(0.7, 0.5) = 0.7$
- **Probabilistic Sum**: $S(0.7, 0.5) = 0.7 + 0.5 - (0.7 \times 0.5) = 0.85$
- **Łukasiewicz T-conorm**: $S(0.7, 0.5) = \min(1, 0.7 + 0.5) = 1$

Applications of T-Norms and T-Conorms

- **Fuzzy Logic Systems**: Used in rule evaluation (AND → T-norm, OR → T-conorm).
- **Fuzzy Control**: Determines the fuzzy outputs in controllers.
- **Decision Making**: Helps aggregate different criteria in multi-attribute decision-making.
- **Image Processing**: Used in edge detection, filtering, and segmentation.