- A **belief** is a probabilistic representation of the robot's state (e.g., position and orientation) given its knowledge and observations.
- Denoted as:

$$bel(x_t) = p(x_t \mid z_{1:t}, u_{1:t})$$

Where:

- x_t: Robot's state at time t
- $z_{1:t}$: Observations from time 1 to t
- $u_{1:t}$: Control inputs from time 1 to t

Types of Belief Representations

a. Histogram Filters

- Represent belief using a grid (discrete bins).
- Each bin holds a probability.
- Simple but memory-intensive; limited resolution.

b. Kalman Filters

- Represent belief as a **Gaussian distribution** (mean + covariance).
- Efficient for linear systems with Gaussian noise.
- Assumes unimodal belief (not suitable for global localization with multiple hypotheses).

c. Extended Kalman Filter (EKF)

- Linearizes non-linear models around the current estimate.
- Works well for small non-linearities.
- Used in landmark-based localization.

d. Particle Filters (Monte Carlo Localization - MCL)

- Represent belief using a set of weighted samples (particles).
- Each particle represents a possible pose.
- Good for non-linear, non-Gaussian systems.
- Handles **multi-modal distributions** well (e.g., global localization, kidnapped robot problem).

Belief Update Process

Bayes Filter Framework

a. Prediction (Motion Update)

$$\overline{bel}(x_t) = \int p(x_t \mid u_t, x_{t-1}) \cdot bel(x_{t-1})\, dx_{t-1}$$

- Incorporates motion model and control input.

b. Correction (Measurement Update)

$$bel(x_t) = \eta \cdot p(z_t \mid x_t) \cdot \overline{bel}(x_t)$$

- Incorporates sensor measurements.
- η is a normalization constant.

Challenges in Belief Representation

- Trade-off between **accuracy and computational cost**.
- Maintaining belief in **dynamic or ambiguous environments**.
- **Sensor fusion**: combining data from multiple sources (e.g., LiDAR, GPS, odometry).
- **Resampling in particle filters** can lead to particle depletion.

Applications

- Indoor robot navigation (e.g., service robots, warehouse robots).
- Autonomous vehicles.
- Drone localization in GPS-denied environments.