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- ***Quadric Surface:*** It is described with Second Degree Equation (quadratics). Quadric surfaces, particularly spheres and ellipsoids, are common elements of graphics scenes.
    - *Spheres*: In Cartesian coordinates, a spherical surface with radius r centered on the coordinate origin is defined as the set of points (x, y, z) that satisfy the equation

$$x^2 + y^2 + z^2 = r^2$$

- ***Ellipsoid:*** An ellipsoidal surface can be described as an extension of a spherical surface, where the radii in three mutually perpendicular directions can have different values.
- ***Blobby Objects:*** Irregular surfaces and change their surface characteristics in certain motion. Called blobby objects. e.g cloth, rubber melting object water droplets etc.

Concept of Spline

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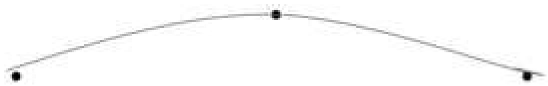
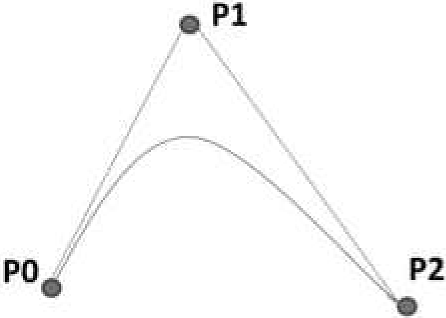
- **Spline:** Spline means a flexible strip used to produce a smooth curve through a designated set of points. A Spline curve specify by a giving set of coordinate positions called ***"Control Point".***
  - *Interpolation* is when a curve passes through a set of "*control points*."

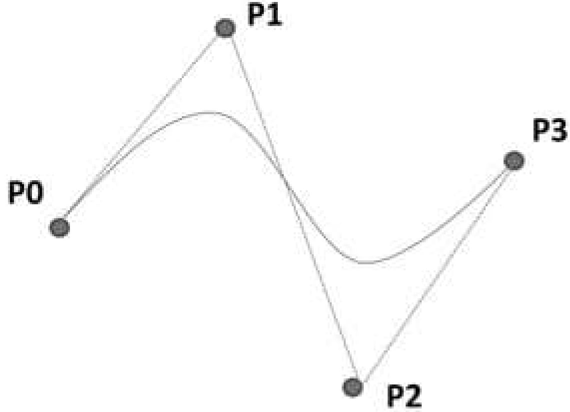


  - *Approximation* is when a curve approximates but doesn't necessarily contain its *control points*.

Bezier Curve

- **Bezier Curve:** Bezier curves are defined using four control points, known as *knots*. Two of these are the end points of the curve, while the other two points control the shape of the curve





Bezier Curve

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Suppose the Curve has n + 1 control-point positions:

pk = (x_k, y_k, z_k), with k varying from 0 to n. These coordinate points can be blended to produce the following position vector P(u), which describes the path of an approximating Bezier polynomial function between p0 and p_n. The position of vector can be given

$$\mathbf{P(u)} = \sum_{k=0}^{n} P_k \, BEZ_{k,n}(u) \qquad 0 \leq u \leq 1$$

Bezier Curve

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The Bezier blending functions $BEZ_{k,n}(u)$ are the Bemstein polynomials:

$$BEZ_{k,n}(u) = C(n,k)\, u^k (1-u)^{n-k}$$

where the ***C(n, k)*** are the binomial coefficients

$$C(n,k) = \frac{n!}{k!\,(n-k)!}$$

The Bezier blending functions with the recursive calculation

$$BEZ_{k,n}(u) = (1-u)\, BEZ_{k,n-1}(u) + u\, BEZ_{k-1,n-1}(u) \quad ,n>k \geq 1$$

Where, $BEZ_{k,k} = u^k$, and $BEZ_{0,k} = (1-u)^k$.

Bezier Curve

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A set of three parametric equations for the individual curve coordinates.

$$x(u) = \sum_{k=0}^{n} x_k BEZ_{k,n}(u)$$

$$y(u) = \sum_{k=0}^{n} y_k BEZ_{k,n}(u)$$

$$z(u) = \sum_{k=0}^{n} z_k BEZ_{k,n}(u)$$

Bezier Curve

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- ***Properties of Bezier Curve:***
  1. Bezier curve always passes through the first & last control points.
  2. The degree of polynomial defining the curve segment is one less than the number of defining polygon points. Therefore for 4, control points the degree of polynomial is three.
  3. The curve generally follows the shape of the defining polygon.
  4. The curve lies entirely within the convex hull formed by control points.
  5. The curve exhibits the variation diminishing property. This means that the curve does not oscillate about any straight-line move often than the defining polygon
  6. The curve is invariant under an affine transformation.

Bezier Curve - Disadvantage

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It has **two main disadvantages**.

- The number of control points is directly related to the degree. Therefore, to increase the complexity of the shape of the curve by adding control points requires increasing the degree of the curve or satisfying the continuity conditions between consecutive segments of a composite curve.
- Changing any control points affects the entire curve or surface, making design of specific sections very difficult.

Bezier Curve- Numerical

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Q1. Construct the bezier curve of order 3 and with 4 polygon vertices A(1,1) , B(2,3) , C(4, 3),D(6,4).

Solution: the equation of bezier curve is given as

$$\mathbf{P(u)} = \sum_{k=0}^{3} P_k \, BEZ_{k,n}(u)$$

$$P(u) = (1-u)^3 \text{P1} + 3u(1-u)^2 \, P2 + 3u^2(1-u)P3 + u^3 \text{P4}$$

for $0 \le u \le 1$

Where P(u) is the point on curve

Let us take **u = 0, ¼, ½, ¾, 1**

Bezier Curve- Numerical

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At u=0, P(0) = (1,1)

At u =1/4

P(1/4)=$(1-1/4)^3$P1+$3*1/4(1-1/4)^2 P2 + 3*(\frac{1}{4})^2(1-1/4)P3 + (\frac{1}{4})^3$

$= \left(1-\frac{1}{4}\right)^3 (1,1)+3*1/4(1-1/4)^2(2,3) + 3*\left(\frac{1}{4}\right)^2\left(1-\frac{1}{4}\right)(4,3) + (\frac{1}{4})^3(6,4)$

= 27/64(1,1)+ 27/64 (2,3) + 9/64 (4,3) + 1/64(6,4)

= [27/64 *1 +27/64 * 2 + 9/64 *4 + 1/64 *6, 27/64 *1 +27/64 * 3 + 9/64 *3 + 1/64 *4]

= [123/64, 139/64]

= (1.9218 , 2.1718)

At u= ½ , P(1/2) = (3.125, 2.87)

At u = ¾, P(3/4) = (4.5156, 3.375)

At u= 1 , P(1) = (6,4)

These are control points for Bezier Curve.

B-Spline Curve

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The disadvantages are remedied with the *B-spline (basis- spline)* representation. The coordinate positions along a B-spline curve in a blending-function formulation as

$$P(u) = \sum_{k=0}^{n} p_k B_{k,d}(u),\ u_{min} \le \mathbf{u} \le u_{max}$$

Where p_k are an input set of n + 1 control points.

B-Spline Curve

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Blending functions for B-spline curves are defined by

$$B_{k,1}(u) = \begin{cases} 1 \ , if\ u_k \le u \le u_{k+1} \\ 0\ ,\ Otherwise \end{cases}$$

$$B_{k,d}(u) = \frac{u-u_k}{u_{k+d-1}} B_{k,d-1}(u) + \frac{u_{k+d}-u}{u_{k+d}-u_{k+d}} B_{k+1,d-1}(u)$$

Where, each blending function is defined over **d** subintervals of the total range of ***u***. The selected set of subinterval endpoints u, is referred to as a **knot** vector.

B-Spline Curve

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Types of Knot vector:

1. Uniform Knot: In a Uniform Knot vector individual knot values are evenly spaced e.g. [0 1 2 3 4].

2. Open Uniform Knot: It has multiplicity of knot values at the ends equal to the order k of B-Spline basis function. Internal knot values are evenly spaced.

e.g. k=2 [0 0 1 2 **3 3**]

k= 3[0 0 0 1 2 **3 3 3**]

k= 4[0 0 0 0 1 **2 2 2 2**]

B-Spline Curve

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Properties of B-spline curve:

1. The degree of B-Spline polynomial is independent of the number of vertices of the polygon.
2. The curve generally follows the shape of the defining polygon.
3. The curve lies within the convex hull of its defining polygon.
4. The curve exhibits the variation diminishing property. This means that the curve does not oscillate about any straight-line move often than the defining polygon.
5. The B-Spline allows local control over the curve surface because each vertex affects the shape of the curve only over a range of parameter values where its associated basic function is non-zero.


Thank you!