UNIT 5NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION
EULER'S METHOD

Euler's Method:

$$y_{n+1} = y_n + hF(x_n, y_n), \quad n = 0, 1, 2, \ldots$$

Modified Euler's Method:-

$$y_{n+1} = y_n + h\left[F\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}F(x_n, y_n)\right)\right]$$

1) Using Euler's Method, find the soln of the initial value problem $\frac{dy}{dx} = \log(x+y)$, $y(0) = 2$ at $x = 0.2$ by assuming $h = 0.2$

Given: $F(x, y) = \log(x+y)$

$x_0 = 0$, $y_0 = 2$, $x_1 = 0.2$ $\quad h = x_1 - x_0 = 0.2$

$$y_{n+1} = y_n + hF(x_n, y_n)$$

$$y_1 = y_0 + hF[x_0, y_0]$$

$$= 2 + (0.2)\log(x_0 + y_0)$$

$$= 2 + (0.2)\log(0+2)$$

$$= 2 + (0.2)\log 2$$

$$= 2 + (0.2)(0.3010)$$

$$= 2.0602.$$

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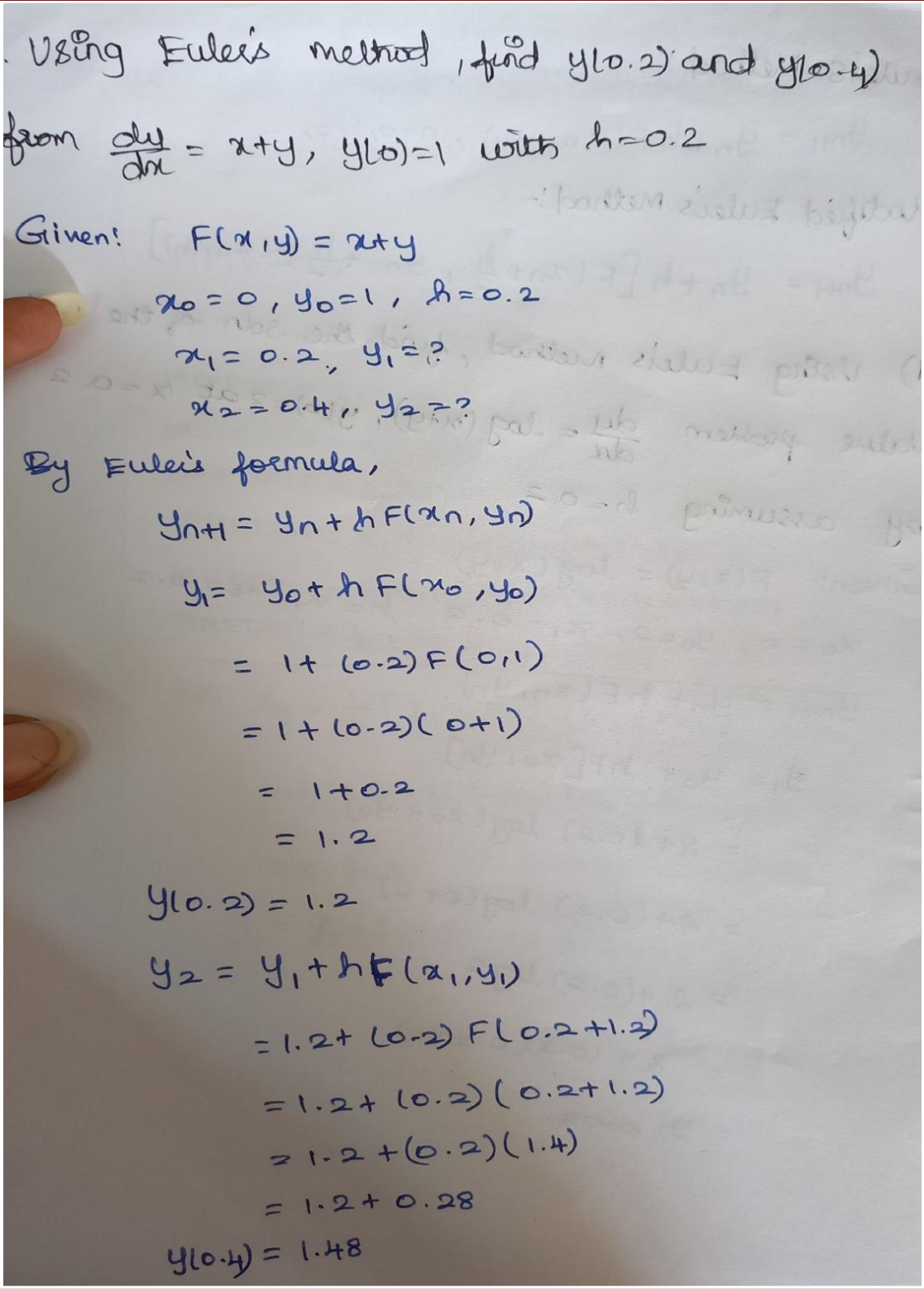

Using Euler's method, find $y(0.2)$ and $y(0.4)$ from $\frac{dy}{dx} = x+y$, $y(0)=1$ with $h=0.2$

Given: $F(x,y) = x+y$

$x_0 = 0, y_0 = 1, h = 0.2$

$x_1 = 0.2, y_1 = ?$

$x_2 = 0.4, y_2 = ?$

By Euler's formula,

$$y_{n+1} = y_n + hF(x_n, y_n)$$

$$y_1 = y_0 + hF(x_0, y_0)$$

$$= 1 + (0.2)F(0,1)$$

$$= 1 + (0.2)(0+1)$$

$$= 1 + 0.2$$

$$= 1.2$$

$y(0.2) = 1.2$

$$y_2 = y_1 + hF(x_1, y_1)$$

$$= 1.2 + (0.2)F(0.2+1.2)$$

$$= 1.2 + (0.2)(0.2+1.2)$$

$$= 1.2 + (0.2)(1.4)$$

$$= 1.2 + 0.28$$

$y(0.4) = 1.48$

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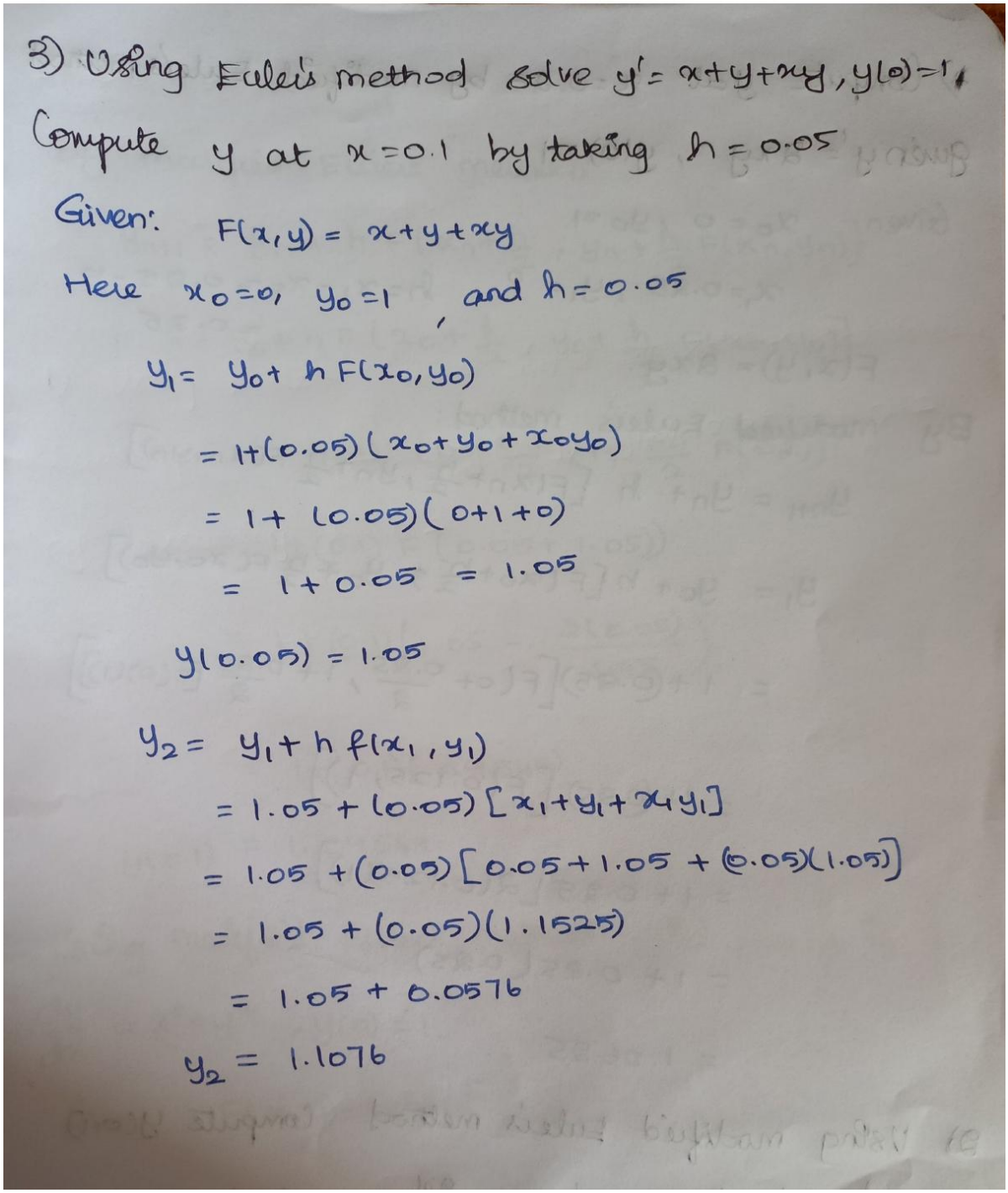

3) Using Euler's method solve $y' = x+y+xy$, $y(0)=1$, Compute y at $x=0.1$ by taking $h=0.05$

Given: $F(x,y) = x+y+xy$

Here $x_0=0$, $y_0=1$, and $h=0.05$

$y_1 = y_0 + h F(x_0, y_0)$

$= 1+(0.05)(x_0+y_0+x_0y_0)$

$= 1+(0.05)(0+1+0)$

$= 1+0.05 = 1.05$

$y(0.05) = 1.05$

$y_2 = y_1 + h f(x_1, y_1)$

$= 1.05 + (0.05)[x_1+y_1+x_1y_1]$

$= 1.05 + (0.05)[0.05+1.05+(0.05)(1.05)]$

$= 1.05 + (0.05)(1.1525)$

$= 1.05 + 0.0576$

$y_2 = 1.1076$