UNIT 5NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION
MODIFIED EULERS METHOD

1) Compute y at $x = 0.25$ by modified Euler's method given $y' = 2xy$, $y(0) = 1$.

Given $x_0 = 0$, $y_0 = 1$

$x_1 = 0.25$, $y_1 = ?$ $\quad h = x_1 - x_0 = 0.25 - 0 = 0.25$

$F(x, y) = 2xy$.

By modified Euler's method:

$$y_{n+1} = y_n + h\left[F\left(x_n + \frac{h}{2}, y_n + \frac{h}{2}F(x_n, y_n)\right)\right]$$

$$y_1 = y_0 + h\left[F\left(x_0 + \frac{h}{2}, y_0 + \frac{h}{2}F(x_0, y_0)\right)\right]$$

$$= 1 + (0.25)\left[F\left(0 + \frac{0.25}{2}, 1 + \frac{0.25}{2}[2(0)(1)]\right)\right]$$

$$= 1 + (0.25)\left[F(0.125, 1)\right]$$

$$= 1 + 0.25\left[2(0.125)(1)\right]$$

$$= 1 + 0.25(0.25)$$

$$= 1.0625$$

2) Using modified Euler's method compute $y(0.1)$ with $h = 0.1$ from $y' = y - \frac{2x}{y}$, $y(0) = 1$.

Given: $x_0 = 0$, $y_0 = 1$, $h = 0.1$

$x_1 = 0.1$, $y_1 = ?$

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$F(x,y) = y - \frac{2x}{y}$

By modified Eulers method,

$$y_{n+1} = y_n + h\,F\left(x_n + \frac{h}{2},\ y_n + \frac{h}{2}F(x_n, y_n)\right]$$

$$y_1 = y_0 + h\,F\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}F(x_0, y_0)\right]$$

$$= 1 + (0.1)F\left(0 + \frac{0.1}{2},\ 1 + \frac{0.1}{2}F(0,1)\right)$$

$$= 1 + (0.1)F(0.05 + 1.05))$$

$$= 1 + (0.1)\left(1.05 - \frac{2(0.05)}{1.05}\right)$$

$$= 1 + (0.1)(0.9548)$$

$$y(0.1) = 1.09548$$

Using modified Euler's method find $y(0.1)$ if

$\frac{dy}{dx} = x^2 + y^2$, $y(0) = 1$.

Given: $F(x,y) = x^2 + y^2$, $y_0 = 1$, $x_0 = 0$.

$x_1 = 0.1$, $y_1 = ?$

$h = x_1 - x_0 = 0.1 - 0 = 0.1$

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Modified Euler's method

$$y_{n+1} = y_n + h\,F\left[x_n + \frac{h}{2},\ y_n + \frac{h}{2}F(x_n, y_n)\right]$$

$$y_1 = y_0 + h\,F\left(x_0 + \frac{h}{2},\ y_0 + \frac{h}{2}F(x_0, y_0)\right]$$

$$= 1 + (0.1)\left[F\left(0 + \frac{0.1}{2},\ 1 + \frac{0.1}{2}F(0,1)\right]\right.$$

$$= 1 + (0.1)\,F\left[0.05, 1 + 0.05(1)\right]$$

$$= 1 + (0.1)\,F(0.05, 1.05)$$

$$= 1 + (0.1)\left[(0.05)^2 + (1.05)^2\right]$$

$$= 1 + (0.1)(1.105)$$

$$= 1 + 0.1105$$

$$= 1.1105$$