



UNIT 5 NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION
FOURTH ORDER RUNGE KUTTA METHOD FOR SOLVING 1ST ORDER EQUATIONS

Given $\frac{dy}{dx} = x^3 + y$, $y(0) = 2$. Compute $y(0.2)$, $y(0.4)$ and $y(0.6)$ by RK method of fourth order.

Given: $y' = x^3 + y$, $F(x, y) = x^3 + y$

Here $x_0 = 0$, $y_0 = 2$, $h = x_1 - x_0 = 2 - 0 = 2$.

$$\begin{aligned}\text{Now, } K_1 &= h F(x_0, y_0) = 0.2 F(0, 2) \\ &= (0.2)(0^3 + 2) \\ &= (0.2)(2) = 0.4\end{aligned}$$

$$\begin{aligned}K_2 &= h F\left[x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right] = (0.2) F\left[0 + \frac{0.2}{2}, 2 + \frac{0.4}{2}\right] \\ &= (0.2) F(0.1, 2.2) = (0.2)[(0.1)^3 + 2.2] \\ &= 0.4402.\end{aligned}$$

$$\begin{aligned}K_3 &= h F\left[x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right] = (0.2) F\left[0 + \frac{0.2}{2}, 2 + \frac{0.4402}{2}\right] \\ &= (0.2) F(0.1, 2.2201) \\ &= (0.2)[(0.1)^3 + 2.2201] \\ &= 0.4442.\end{aligned}$$

$$\begin{aligned}K_4 &= h F[x_0 + h, y_0 + K_3] = (0.2) F[0 + 0.2, 2 + 0.4442] \\ &= (0.2) F(0.2, 2.4442) \\ &= 0.2[(0.2)^3 + 2.4442] \\ &= 0.4904\end{aligned}$$



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$$\Delta y = \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$

$$= \frac{1}{6} [0.4 + 2(0.4402) + 2(0.4442) + 0.4904]$$

$$= 0.4432.$$

$$y_1 = y_0 + \Delta y = 2 + 0.4432 = 2.4432.$$

$$\begin{aligned} \text{Now, } K_1 &= hF(x_1, y_1) = (0.2)F(0.2, 2.4432) \\ &= (0.2)[(0.2)^3 + 2.4432] \\ &= 0.4902. \end{aligned}$$

$$\begin{aligned} K_2 &= hF\left[x_1 + \frac{h}{2}, y_1 + \frac{K_1}{2}\right] \\ &= (0.2)F\left[0.2 + \frac{0.2}{2}, 2.4432 + \frac{0.4902}{2}\right] \\ &= (0.2)F[0.3, 2.6883] \\ &= (0.2)[(0.3)^3 + 2.6883] = 0.5431 \end{aligned}$$

$$\begin{aligned} K_3 &= hF\left[x_1 + \frac{h}{2}, y_1 + \frac{K_2}{2}\right] \\ &= (0.2)F\left[0.2 + \frac{0.2}{2}, 2.4432 + \frac{0.5431}{2}\right] \\ &= (0.2)F[0.3, 2.7148] \\ &= (0.2)[(0.3)^3 + 2.7148] \end{aligned}$$

$$K_3 = 0.5484.$$



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$$K_4 = hF[x_1 + h, y_1 + K_3]$$

$$= 0.2 F[0.2 + 0.2, 2.4432 + 0.5484]$$

$$= (0.2) F[0.4, 2.9916]$$

$$= (0.2) [(0.4)^3 + 2.9916]$$

$$K_4 = 0.6111$$

$$\Delta y = \frac{1}{6} [K_1 + 2K_2 + 2K_3 + K_4]$$

$$= \frac{1}{6} [0.4902 + 2(0.5431) + 2(0.5484) + 0.6111]$$

$$\Delta y = 0.5474$$

$$y_2 = y_1 + \Delta y = 2.4432 + 0.5474$$

$$y_2 = 2.9906.$$

$$\text{Here } x_2 = 0.4, y_2 = 2.9906 \text{ and } h = x_2 - x_1 \\ = 0.4 - 0.2 = 0.2$$

$$\text{Now, } K_1 = hF[x_2, y_2] = (0.2) F[0.4, 2.9906]$$

$$= (0.2) [(0.4)^3 + 2.9906]$$

$$K_1 = 0.6109$$

$$K_2 = hF\left[x_2 + \frac{h}{2}, y_2 + \frac{K_1}{2}\right]$$

$$= (0.2) F\left[0.4 + \frac{0.2}{2}, 2.9906 + \frac{0.6109}{2}\right]$$

$$= (0.2) F[0.5, 3.29605]$$

$$= 0.6842$$