**UNIT 5NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION**
FOURTH ORDER RUNGE KUTTA METHOD FOR SOLVING 1ST ORDER EQUATIONS

2. Using Runge Kutta method of $4^{th}$ order solve

$y' = \dfrac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$

Given: $y' = \dfrac{y^2 - x^2}{y^2 + x^2}$ ; $F(x,y) = \dfrac{y^2 - x^2}{y^2 + x^2}$

Here $x_0 = 0$, $y_0 = 1$, $h = x_1 - x_0 = 0.2 - 0 = 0.2$

$x_1 = 0.2$, $y_1 = ?$

Now $K_1 = hF(x_0, y_0)$

$K_1 = (0.2)\left[\dfrac{(1)^2 - 0}{(1)^2 + 0}\right] = 0.2$

$K_2 = hF\left[x_0 + \frac{h}{2}, y_0 + \frac{K_1}{2}\right]$

$= (0.2)F\left[0 + \frac{0.2}{2}, 1 + \frac{0.2}{2}\right]$

$= (0.2)F[0.1, 1.1] = (0.2)\left[\dfrac{(1.1)^2 - (0.1)^2}{(1.1)^2 + (0.1)^2}\right]$

$= 0.1967$

$K_3 = hF\left[x_0 + \frac{h}{2}, y_0 + \frac{K_2}{2}\right]$

$= (0.2)F\left[0 + \frac{0.2}{2}, 1 + \frac{0.1967}{2}\right]$

$= (0.2)F[0.1, 1.0984]$

$= (0.2)\left[\dfrac{(1.0984)^2 - (0.1)^2}{(1.0984)^2 + (0.1)^2}\right] = (0.2)(0.9836)$

$= 0.1967.$

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$$K_4 = hF(x_0+h,\ y_0+K_3)$$

$$= (0.2)F(0+0.2,\ 1+0.1967)$$

$$= 0.2\left[\frac{(1.1967)^2-(0.2)^2}{(1.1967)^2+(0.2)^2}\right] = (0.2)(0.9457)$$

$$K_4 = 0.1891$$

$$\Delta y = \frac{1}{6}(K_1+2K_2+2K_3+K_4)$$

$$= \frac{1}{6}[0.2+2(0.1967)+2(0.1967)+0.1891]$$

$$= \frac{1}{6}[1.1759] = 0.1960.$$

$$y_1 = y_0+\Delta y = 1+0.1960 = 1.1960.$$

3) Find $y(0.8)$ given that $y' = y-x^2$, $y(0.6) = 1.7379$ by using R.K method of 4th order take $h=0.1$

Given $y' = y-x^2$, $F(x,y) = y-x^2$

Here $x_0 = 0.6$, $y_0 = 1.7379$, $h=0.1$

$x_1 = 0.7$, $y_1 = ?$

$x_2 = 0.8$, $y_2 = ?$

Now $K_1 = hF[x_0, y_0] = (0.1)F[0.6,\ 1.7379] = (0.1)[1.7379-(0.6)^2]$

$$K_1 = 0.1378$$

$$K_2 = hF\left[x_0+\frac{h}{2},\ y_0+\frac{K_1}{2}\right] = (0.1)F\left[0.6+\frac{0.1}{2},\ 1.7379+\frac{0.1378}{2}\right]$$

$$= (0.1)F[0.65,\ 1.8068]$$

$$= (0.1)[1.8068-(0.65)^2]$$

$$= 0.1384$$

$$K_3 = hF\left[x_0+\frac{h}{2},\ y_0+\frac{K_2}{2}\right] = (0.1)F\left[0.6+\frac{0.1}{2},\ 1.7379+\frac{0.1384}{2}\right]$$

$$=(0.1)F[0.65, 1.8071]$$

$$=(0.1)[1.8071-(0.65)^2]$$

$$=0.1385$$

$$K_4 = hF[x_0+h, y_0+K_3]$$

$$=(0.1)F[0.6+0.1,\ 1.7379+0.1385]$$

$$=(0.1)F[0.7, 1.8764] = (0.1)[1.8764-(0.7)^2]$$

$$K_4 = 0.13864$$

$$\Delta y = \frac{1}{6}(K_1+2K_2+2K_3+K_4)$$

$$=\frac{1}{6}(0.1378+2(0.1384)+2(0.1385)+0.1386)$$

$$=\frac{1}{6}(0.8302)$$

$$=0.1384$$

$$y_1 = y_0+\Delta y = 1.7379+0.1384$$

$$=1.8763.$$

Here $x_1 = 0.7,\ y_1 = 1.8763$

$$K_1 = hF(x_1, y_1) = (0.1)F(0.7, 1.8763)$$

$$=(0.1)(1.8763-(0.7)^2)$$

$$=0.1386$$

$$K_2 = hF\left(x_1+\frac{h}{2},\ y_1+\frac{K_1}{2}\right) = (0.1)F\left[0.7+\frac{0.1}{2},\ 1.8763+\frac{0.1386}{2}\right]$$

$$=(0.1)F(0.75, 1.9456) = 0.1[1.9456-(0.75)^2]$$

$$=0.1383$$

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FOURTH ORDER RUNGE KUTTA METHOD FOR SOLVING 1ST ORDER EQUATIONS

$$K_3 = hF\left(x_1+\frac{h}{2}, y_1+\frac{K_2}{2}\right) = (0.1)F\left[0.7+\frac{0.1}{2}, 1.8763+\frac{0.1383}{2}\right]$$

$$= (0.1)F[0.75, 1.9455] = (0.1)[1.9455-(0.75)^2]$$

$$= 0.1383$$

$$K_4 = hF(x_1+h, y_1+K_3) = (0.1)[0.7+0.1, 1.8763+0.1383]$$

$$= (0.1)F[0.8, 2.0146] = (0.1)[2.0146-(0.8)^2]$$

$$= 0.1375$$

$$\Delta y = \frac{1}{6}[K_1+2K_2+2K_3+K_4]$$

$$= \frac{1}{6}[0.1386+2(0.1383)+2(0.1383)+0.1375]$$

$$= 0.1382$$

$$y_2 = y_1+\Delta y = 1.8763+0.1382$$

$$= 2.0145.$$