UNIT 5NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION
MILNE'S PREDICTOR CORRECTOR METHOD FOR SOLVING 1ST ORDER EQUATIONS

1. Solve $y' = x - y^2$, $0 \le x \le 1$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ by Milne's method. To find $y(0.8)$ and $y(1)$

Here, $x_0 = 0$, $y_0 = 0$

$x_1 = 0.2$, $y_1 = 0.02$

$x_2 = 0.4$, $y_2 = 0.0795$

$x_3 = 0.6$, $y_3 = 0.1762$

$x_4 = 0.8$, $y_4 = ?$

$x_5 = 1$, $y_5 = ?$

By Milne's predictor formula,

$$y_{n+1,P} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

put $n=3$,

$$y_{4,P} = y_0 + \frac{4h}{3}\left[2y'_1 - y'_2 + 2y'_3\right]$$

Given: $y' = x - y^2$

$$y_1' = x_1 - y_1^2 = (0.2) - (0.02)^2 = 0.1996$$

$$y_2' = x_2 - y_2^2 = (0.4) - (0.0795)^2 = 0.3937$$

$$y_3' = x_3 - y_3^2 = 0.6 - (0.1762)^2 = 0.5690$$

$$y_{4,p} = 0 + \frac{4(0.2)}{3}\left[2(0.1996) - 0.3939 + 2(0.569)\right]$$

$$= 0.3049$$

$$y_4' = x_4 - y_4^2 = (0.8) - (0.3049)^2 = 0.707$$

$$\therefore\ y_{4,c} = y_2 + \frac{h}{3}\left[y_2' + 4y_3' + y_4'\right]$$

$$= 0.0795 + \frac{0.2}{3}\left[0.3939 + 4(0.5690) + 0.707\right]$$

$$= 0.3046$$

$\therefore$ Corrected value of y at $x = 0.8$ is 0.3046

To find $y(1)$

$n = 4$, $$y_{5,p} = y_1 + \frac{4h}{3}\left[2y_2' - y_3' + 2y_4'\right]$$

$$= 0.02 + \frac{4(0.2)}{3}\left[2(0.3937) - 0.5690 + 2(0.707)\right]$$

$$= 0.4553$$

$$y_5' = x_5 - y_5^2 = 1 - (0.4553)^2 = 0.7327$$

$$y_{5,c} = y_3 + \frac{h}{3}\left[y_3' + 4y_4' + y_5'\right]$$

$$= 0.1762 + \frac{0.2}{3}\left[0.569 + 4(0.707) + 0.7327\right]$$

$$= 0.4515$$

$\therefore$ The corrected value of y at $x = 1$ is 0.4515