



UNIT 5 NUMERICAL SOLUTION OF ORDINARY DIFFERENTIAL EQUATION
MILNE'S PREDICTOR CORRECTOR METHOD FOR SOLVING 1ST ORDER EQUATIONS

2) Given that $\frac{dy}{dx} = \frac{1}{2}(1+x^2)y^2$; $y(0)=1$, $y(0.1)=1.06$,
 $y(0.2)=1.12$ and $y(0.3)=1.21$. Evaluate $y(0.4)$ &
 $y(0.5)$ by milne's predictor corrector method.

Given: $\frac{dy}{dx} = y' = \frac{1}{2}(1+x^2)y^2$ and $h=0.1$

Here $x_0 = 0$, $y_0 = 1$

$x_1 = 0.1$, $y_1 = 1.06$

$x_2 = 0.2$, $y_2 = 1.12$

$x_3 = 0.3$, $y_3 = 1.21$

$x_4 = 0.4$, $y_4 = ?$

$x_5 = 0.5$, $y_5 = ?$

By Milne's predictor formula, we have

$$y_{n+1,p} = y_{n-3} + \frac{4h}{3} [2y'_{n-2} - y'_{n-1} + 2y'_n] \rightarrow ①$$

Put $n=3$,

$$y_{4,p} = y_0 + \frac{4h}{3} [2y'_1 - y'_2 + 2y'_3] \rightarrow ②$$

$$\text{Now, } y'_1 = \frac{1}{2}(1+x_1^2)y_1^2 = \frac{1}{2}(1+(0.1)^2)(1.06)^2 \\ = 0.5674$$

$$y'_2 = \frac{1}{2}(1+x_2^2)y_2^2 = \frac{1}{2}(1+(0.2)^2)(1.12)^2 = 0.6523$$

$$y'_3 = \frac{1}{2}(1+x_3^2)y_3^2 = \frac{1}{2}(1+(0.3)^2)(1.21)^2 = 0.7979$$



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$$\textcircled{2} \Rightarrow y_{4,p} = 1 + \frac{4(0.1)}{3} [2(0.5674) - 0.6523 + 2(0.7979)]$$

$$= 1 + \frac{0.4}{3} [2.0783] = 1 + 0.2771 = 1.2771$$

$$y(0.4) = 1.2771$$

$$y_4' = \frac{1}{2}(1+x_4^2)(y_4^2) = \frac{1}{2}(1+(0.4)^2)(1.2771)^2 = 0.9460$$

By Milne's corrector formula,

$$y_{n+1,c} = y_{n-1} + \frac{h}{3} [y_{n-1}' + 4y_n' + y_{n+1}']$$

$$\text{put } n=3, y_{4,c} = y_2 + \frac{h}{3} [y_2' + 4y_3' + y_4']$$

$$= 1.12 + \frac{0.1}{3} (0.653 + 4(0.7979) + 0.946)$$

$$= 1.12 + 0.1597$$

$$y(0.4) = 1.2797$$

To find $y(0.5)$

To get y_5 , put $n=4$ in $\textcircled{1}$ we get

$$y_{5,p} = y_1 + \frac{4h}{3} (2y_2' - y_3' + 2y_4')$$

$$\text{Now } y_4' = \frac{1}{2}(1+x_4^2)y_4^2 = \frac{1}{2}(1+(0.4)^2)(1.2797)^2 = 0.9498$$

$$y_{5,p} = 1.06 + \frac{4(0.1)}{3} [2(0.6523) - 0.7979 + 2(0.9498)]$$

$$= 1.06 + \frac{0.4}{3} (2.4062)$$

$$y(0.5) = 1.3808$$



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By Milne's corrector formula, we have

$$y_{n+1,c} = y_{n+1} + \frac{h}{3} [y'_{n-1} + 4y'_n + y'_{n+1}]$$

put $n=4$,

$$y_{5,c} = y_3 + \frac{h}{3} [y'_3 + 4y'_4 + y'_5] \rightarrow \textcircled{6}$$

$$y'_5 = \frac{1}{2} (1+x^2) y^2$$

$$= \frac{1}{2} (1+(0.5)^2) (1.3808)^2 = 1.1917$$

$$y_{5,c} = 1.21 + \frac{0.1}{3} (0.7979 + 4(0.9498) + 1.1917)$$

$$= 1.21 + \frac{0.1}{3} (5.7888)$$

$$= 1.21 + 0.193$$

$$y(0.5) = 1.403$$