DERIVATIVES FROM DIFFERENCE TABLES – DIVIDED DIFFERENC AND FINITE DIFFERENCES :

$$\frac{dy}{dx} = \frac{1}{h}\left[\Delta y_0 + \frac{2u-1}{2!}\Delta^2 y_0 + \frac{3u^2-6u+2}{3!}\Delta^3 y_0 + \frac{4u^3-18u^2+22u-6}{4!}\Delta^4 y_0 + \cdots\right] \quad \text{—— (}$$

putting $x = x_0$, then $u=0$ and above eqn. reduces to

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\Delta^2 y_0 + \frac{6u-6}{3!}\Delta^3 y_0 + \frac{12u^2-36u+22}{4!}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\frac{6}{3!}\Delta^3 y_0 + \frac{24u-36}{4!}\Delta^4 y_0 + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_0} = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \frac{7}{4}\Delta^5 y_0\right]$$

NEWTON'S BACKWARD DIFFERENCE FORMULA:

$$\frac{dy}{dx} = \frac{1}{h}\left[\nabla y_n + \frac{2u+1}{2!}\nabla^2 y_n + \frac{3u^2+6u+2}{3!}\nabla^3 y_n + \frac{4u^3+18u^2+22u+6}{4!}\nabla^4 y_n + \cdots\right]$$

At $x = x_n$, $u = 0$

$$\left(\frac{dy}{dx}\right)_{x=x_n} = \frac{1}{h}\left[\nabla y_n + \frac{1}{2}\nabla^2 y_n + \frac{1}{3}\nabla^3 y_n + \frac{1}{4}\nabla^4 y_n + \cdots\right]$$

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}\left[\nabla^2 y_n + \frac{6u+6}{3!}\nabla^3 y_n + \frac{12u^2+36u+22}{4!}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\nabla^2 y_n + \nabla^3 y_n + \frac{11}{12}\nabla^4 y_n + \cdots\right]$$

$$\frac{d^3y}{dx^3} = \frac{1}{h^3}\left[\frac{6}{3!}\nabla^3 y_n + \frac{24u+36}{4!}\nabla^4 y_n + \cdots\right]$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=x_n} = \frac{1}{h^3}\left[\nabla^3 y_n + \frac{3}{2}\nabla^4 y_n + \frac{7}{4}\nabla^5 y_n\right]$$

1) Find $f'(3)$ and $f''(3)$ for the following data:

x	3.0	3.2	3.4	3.6	3.8	4.0
$f(x)$	−14	−10.032	−5.296	−0.256	6.672	14

x	y	Δy	$\Delta^2 y$	$\Delta^3 y$	$\Delta^4 y$	$\Delta^5 y$
3.0	−14					
		3.968				
3.2	−10.032		0.768			
		4.736		−0.464		
3.4	−5.296		0.304		2.048	
		5.04		1.584		−5.12
3.6	−0.256		1.888		−3.072	
		6.928		−1.488		
3.8	6.672		0.4			
		7.328				
4.0	14					

By Newton's forward formula

$$\left(\frac{dy}{dx}\right)_{x=x_0} = \left(\frac{dy}{dx}\right)_{u=0}$$

$$= \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \frac{1}{4}\Delta^4 y_0 + \frac{1}{5}\Delta^5 y_0 - \cdots\right]$$

Here $h = 0.2$

$$= \frac{1}{0.2}\left[3.968 - \frac{1}{2}(0.768) + \frac{1}{3}(-0.464) - \frac{1}{4}(2.048) + \frac{1}{5}(-5.12)\right]$$

$$= \frac{1}{0.2}\left[3.968 - 0.384 - 0.1547 - 0.512 - 1.024\right]$$

$$= \frac{1}{0.2}\left[1.8933\right]$$

$$= 9.4665$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_0} = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 - \frac{5}{6}\Delta^5 y_0 + \cdots\right]$$

$$= \frac{1}{(0.2)^2}\left[0.768 - (-0.464) + \frac{11}{12}(2.048) - \frac{5}{6}(-5.12)\right]$$

$$= \frac{1}{0.04}\left[0.768 + 0.464 + 1.8773 + 4.267\right]$$

$$= \frac{1}{0.04}\left[7.3763\right] = \cancel{184.40757} \cdot 36.875$$