



DEPARTMENT OF MATHEMATICS

UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

NUMERICAL INTEGRATION BY SIMPSON'S $\frac{1}{3}$

SIMPSON'S $\frac{1}{3}$ RULE:

$$\int_{x_0}^{x_n} y \, dx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_3 + \dots + y_{n-3}) + 2(y_2 + y_4 + \dots + y_{n-2})]$$

$$= \frac{h}{3} [A + 4B + 2C]$$

where A = Sum of the first & last ordinates

B = Sum of the odd ordinates

C = Sum of the even ordinates

(ie) an even number of equal sub-intervals.

Dividing the range into 10 equal parts, find the value

of $\int_0^{\pi/2} \sin x \, dx$ by Simpson's $\frac{1}{3}$ rule



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Soln:

| | | | | | | |
|--------------|-----------|-----------|-----------|-----------|------------|-----------|
| x | 0 | $\pi/20$ | $2\pi/20$ | $3\pi/20$ | $4\pi/20$ | $5\pi/20$ |
| $y = \sin x$ | 0 | 0.1564 | 0.3090 | 0.4540 | 0.5878 | 0.7071 |
| x | $6\pi/20$ | $7\pi/20$ | $8\pi/20$ | $9\pi/20$ | $10\pi/20$ | |
| $y = \sin x$ | 0.8090 | 0.8910 | 0.9511 | 0.9877 | 1 | |

By Simpson's $1/3$ rule,

$$\begin{aligned} \int_0^{\pi/2} \sin x \, dx &= \frac{h}{3} [(y_0 + y_{11}) + 4(y_1 + y_3 + y_5 + y_7 + y_9) + 2(y_2 + y_4 + y_6 + y_8 + y_{10})] \\ &= \frac{\pi}{20} \cdot \frac{1}{3} [(0 + 1) + 4(3.1962) + 2(2.6569)] \\ &= 1.0000 \end{aligned}$$

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Find the value of $\log_e 5$ from $\int_0^5 \frac{dx}{4x+5}$ by Simpson's $\frac{1}{3}$ rule ($n=10$).

Soln: Here $y(x) = \frac{1}{4x+5}$

$$h = \frac{5-0}{10} = \frac{1}{2} = 0.5$$

| | | | | | | | | | |
|-------|-----|--------|--------|--------|--------|--------|--------|--------|-------|
| x : | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 | 3.5 | 4 |
| y : | 0.2 | 0.1429 | 0.1111 | 0.0909 | 0.0769 | 0.0667 | 0.0588 | 0.0526 | 0.047 |
| | | | 4.5 | 5 | | | | | |
| | | | 0.0434 | 0.04 | | | | | |

By Simpson's $\frac{1}{3}$ rule,

$$\int_0^5 \frac{dx}{4x+5} = \frac{h}{3} [(y_0 + y_n) + 2(y_2 + y_4 + y_6 + \dots) + 4(y_1 + y_3 + y_5 + \dots)]$$



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$$= \frac{1}{6} [2.4148]$$

$$= 0.4025 \text{ --- (1)}$$

$$\int_0^5 \frac{dx}{4x+5} = \frac{\log(4x+5)}{4} \Big|_0^5$$

$$= \frac{1}{4} (\log 25 - \log 5)$$

$$= \frac{1}{4} \log \left(\frac{25}{5} \right)$$

$$= \frac{1}{4} \log 5 \text{ --- (2)}$$

From (1) & (2)

$$\Rightarrow \frac{1}{4} \log 5 = 0.4025$$

$$\Rightarrow \log 5 = 1.61$$

log e