MILNE'S PREDICTOR AND CORRECTOR METHODS

FORMULAS:

$$y_{n+1,P} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right] \rightarrow \text{Milne's predictor formulae}$$

$$y_{n+1,C} = y_{n-1} + \frac{h}{3}\left[y'_{n-1} + 4y'_n + y'_{n+1}\right] \rightarrow \text{Milne's Corrector formulae}$$

Solve $y' = x - y^2$, $0 \le x \le 1$, $y(0) = 0$, $y(0.2) = 0.02$, $y(0.4) = 0.0795$, $y(0.6) = 0.1762$ by Milne's method to find $y(0.8)$ and $y(1)$.

Soln:

Gn:

$x_0 = 0 \rightarrow y_0 = 0$

$x_1 = 0.2 \rightarrow y_1 = 0.02$

$x_2 = 0.4 \rightarrow y_2 = 0.0795$

$x_3 = 0.6 \rightarrow y_3 = 0.1762$

$x_4 = 0.8 \rightarrow y_4 = ?$

$x_5 = 1 \rightarrow y_5 = ?$

Here $h = 0.2$.

WKT Milne's predictor formula is

$$y_{n+1,P} = y_{n-3} + \frac{4h}{3}\left[2y'_{n-2} - y'_{n-1} + 2y'_n\right]$$

$$y_{4,P} = y_0 + \frac{4h}{3}\left[2y_1' - y_2' + 2y_3'\right]$$

Gn: $y' = x - y^2$

$$y_1' = x_1 - y_1^2 = 0.2 - (0.02)^2 = 0.1996$$

$$y_2' = x_2 - y_2^2 = 0.4 - (0.0795)^2 = 0.3937$$

$$y_3' = x_3 - y_3^2 = 0.6 - (0.1762)^2 = 0.5690$$

$$y_{4,P} = 0 + \frac{4(0.2)}{3}\left[2\times 0.1996 - 0.3937 + 2\times 0.5690\right]$$

$$= 0.3049$$

$$y_4' = x_4 - y_4^2 = 0.8 - (0.3049)^2 = 0.707.$$

$$y_{4,c} = y_2 + \frac{h}{3}[y_2' + 4y_3' + y_4']$$

$$= 0.0795 + \frac{0.2}{3}[0.3937 + 4 \times 0.5690 + 0.707]$$

$$= 0.3046$$

$\therefore$ Corrected value of y at $x = 0.8$ is 0.3046.

To find $y(1)$:

$$y_{5,p} = y_1 + \frac{4h}{3}[2y_2' - y_3' + 2y_4']$$

$$= 0.02 + 4 \times \frac{0.2}{3}[2 \times 0.3937 - 0.5690 + 2 \times 0.707)$$

$$= 0.4553$$

$$y_5' = x_5 - y_5^2 = 1 - (0.4553)^2 = 0.7327$$

$$y_{5,c} = y_3 + \frac{h}{3}[y_3' + 4y_4' + y_5']$$

$$= 0.1762 + \frac{0.2}{3}[0.569 + 4 \times 707 + 0.7327]$$

$$= 0.4515$$

$\therefore$ Corrected value of y at $x = 1$ is 0.4515.

(1) Using Milne's method find $y(4.4)$ gn. $5xy' + y^2 - 2 = 0$ given $y(4) = 1$, $y(4.1) = 1.0049$, $y(4.2) = 1.0097$ and $y(4.3) = 1.0143$.

Soln: $y_{4,p} = 1.01897$; $y_{4,c} = 1.01874$

(2) Using Runge-Kutta method calculate $y(0.1)$, $y(0.2)$ and $y(0.3)$ gn that $\frac{dy}{dx} - \frac{2xy}{1+x^2} = 1$, $y(0) = 0$. Taking these values as starting values find $y(0.4)$ by Milne's method

Soln:
$y(0.1) = 0.1006$
$y(0.2) = 0.2052$ } By Rk method.
$y(0.3) = 0.3176$

$y(0.4) = 0.4413$