



DEPARTMENT OF MATHEMATICS

UNIT-IV INTERPOLATION, NUMERICAL DIFFERENTIATION & INTEGRATION

NEWTON'S FORWARD AND BACKWARD DIFFERENCE FORMULA

(EQUAL INTERVALS)

Let the function $y = f(x)$ takes the values y_0, y_1, \dots, y_n at the points x_0, x_1, \dots, x_n where $x_i = x_0 + ih$.

Then Newton's Forward interpolation polynomial is given by

$$y(x) = P_n(x) = f(x)$$

$$= y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 + \dots + \frac{u(u-1)(u-2) \dots (u-(n-1))}{n!} \Delta^n y_0$$

where $u = \frac{x - x_0}{h}$; h is the difference between two intervals.



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Then Newton's Backward interpolation polynomial is given by

$$\begin{aligned} y(x) = P_n(x) &= f(x) \\ &= y_n + \frac{u}{1!} \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n \\ &\quad + \dots + \frac{u(u+1)(u+2)\dots(u+(n-1))}{n!} \nabla^n y_n \end{aligned}$$

where $u = \frac{x - x_n}{h}$

Note: Forward

First order:

$$\Delta y_0 = y_1 - y_0$$

$$\Delta y_1 = y_2 - y_1$$

Second order:

$$\Delta^2 y_0 = \Delta y_1 - \Delta y_0$$

Third order:

$$\Delta^3 y_0 = \Delta^2 y_1 - \Delta^2 y_0$$

Backward

First order:

$$\nabla y_n = y_n - y_{n-1}$$

$$\nabla y_{n-1} = y_{n-1} - y_{n-2}$$

Second order:

$$\nabla^2 y_n = \nabla y_n - \nabla y_{n-1}$$

Third order:

$$\nabla^3 y_n = \nabla^2 y_n - \nabla^2 y_{n-1}$$



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Using Newton's Forward Interpolation & Backward Interpolation formula, find the polynomial $f(x)$ satisfying the following data. Hence evaluate y at $x=5$.

| | | | | |
|-----|---|---|---|----|
| x | 4 | 6 | 8 | 10 |
| y | 1 | 3 | 8 | 10 |

soln:

| x | y | Δy | $\Delta^2 y$ | $\Delta^3 y$ |
|-----|-----|--------------|---------------|---------------|
| 4 | 1 | $(3-1)$ 2 | $(5-2)$ 3 | $(10-8)$ 2 |
| 6 | 3 | $(8-3)$ 5 | $(10-8)$ 2 | |
| 8 | 8 | | | |
| 10 | 10 | | | |



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Forward Interpolation:

Here $x_0 = 4$; $y_0 = 1$; $h = 2$.

$$u = \frac{x-4}{2}$$

$$\begin{aligned} y(x) &= y_0 + \frac{u}{1!} \Delta y_0 + \frac{u(u-1)}{2!} \Delta^2 y_0 + \frac{u(u-1)(u-2)}{3!} \Delta^3 y_0 \\ &= 1 + \left(\frac{x-4}{2}\right) (2) + \left(\frac{x-4}{2}\right) \left(\frac{x-4}{2} - 1\right) \frac{(3)}{2!} + \\ &\quad \left(\frac{x-4}{2}\right) \left(\frac{x-4}{2} - 1\right) \left(\frac{x-4}{2} - 2\right) \frac{(-6)}{3!} \\ &= 1 + x - 4 + \frac{(x-4)(x-6)}{4} \times \frac{3}{2} + \frac{(x-4)(x-6)(x-8)}{8} \times \frac{-6}{8} \\ &= x - 3 + (x^2 - 10x + 24) \frac{3}{8} + x^3 - 18x^2 + 104x - 192 \times \frac{1}{8} \\ &= \frac{1}{8} (8x - 24 + 3x^2 - 30x + 72 + (-x^3 + 18x^2 - 104x + 192)) \\ &= \frac{1}{8} (-x^3 + 21x^2 - 126x + 240) \\ y(5) &= \frac{1}{8} (-5^3 + 21(5)^2 - 126(5) + 240) = 1.25 \end{aligned}$$

Backward Interpolation:

Here $x_n = 10$; $y_n = 10$; $h = 2$.

$$u = \frac{x-10}{2}$$



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$$\begin{aligned} y(x) &= y_n + \frac{u}{1!} \nabla y_n + \frac{u(u+1)}{2!} \nabla^2 y_n + \frac{u(u+1)(u+2)}{3!} \nabla^3 y_n \\ &= 10 + \left(\frac{x-10}{2}\right)(2) + \left(\frac{x-10}{2}\right)\left(\frac{x-10}{2}+1\right)\left(\frac{-3}{2}\right) + \\ &\quad \left(\frac{x-10}{2}\right)\left(\frac{x-10}{2}+1\right)\left(\frac{x-10}{2}+2\right)\left(\frac{-6}{3!}\right) \end{aligned}$$