MODIFIED EULER METHOD:

$$y_1 = y_0 + h\left[f\left(x_0 + \frac{1}{2}h,\ y_0 + \frac{h}{2} f(x_0, y_0)\right)\right] \text{ for } (x_0, y_0)$$

$$y_2 = y_1 + h\left[f\left(x_1 + \frac{h}{2},\ y_1 + \frac{h}{2} f(x_1, y_1)\right)\right] \text{ for } (x_1, y_1)$$

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$$y_{n+1} = y_n + h\left[f\left(x_n + \frac{h}{2},\ y_n + \frac{h}{2} f(x_n, y_n)\right)\right] \text{ for } (x_n, y_n)$$

for $n = 0, 1, 2, \ldots$

This formula is called modified Euler's formula.

Compute y at $x = 0.25$ by modified Euler method given

$y' = 2xy$, $y(0) = 1$

Gn: $f(x, y) = 2xy$

$x_0 = 0$; $y_0 = 1$; $h = 0.25$

$$y_1 = y_0 + h f\left[x_0 + \frac{h}{2},\ y_0 + \frac{h}{2} f(x_0, y_0)\right]$$

$$= 1 + (0.25) f\left[0 + \frac{0.25}{2},\ 1 + \frac{0.25}{2}[2x_0 y_0]\right]$$

$$= 1 + (0.25) f(0.125,\ 1)$$

$$= 1 + 0.25[2(0.125)(1)]$$

$$= 1 + 0.0625$$

$$= 1.0625$$

2) Solve $y' = 1 - y$, $y(0) = 0$ by modified Euler's method with $x_1 = 0.1$, $x_2 = 0.2$, $x_3 = 0.3$.

Soln: $y(0.1) = 0.095$

$y(0.2) = 0.1809$

$y(0.3) = 0.2587$