FOURTH ORDER RUNGE-KUTTA METHOD FOR SOLVING FIRST AND SECOND ORDER EQUATIONS.

SECOND ORDER RK METHOD:

If the initial values of (x,y) for the differential eqn. $\frac{dy}{dx} = f(x,y)$ then the first increment in y namely Δy is calculated from the formula

$$k_1 = hf(x,y)$$

$$k_2 = hf\left[x+\frac{h}{2}, y+\frac{k_1}{2}\right]$$

$$\Delta y = k_2 \quad \text{where } h = \Delta x$$

Now $y(x+h) = y(x) + \Delta y$ (ie) $y_1 = y_0 + \Delta y$

THIRD ORDER RK METHOD:

$$k_1 = hf(x,y)$$

$$k_2 = hf\left[x+\frac{h}{2}, y+\frac{k_1}{2}\right]$$

$k_3 = hf[x+h,\ y+2k_2-k_1]$

$\Delta y = \frac{1}{6}[k_1+4k_2+k_3]$

Now $y_1 = y_0 + \Delta y$

FOURTH ORDER RK METHOD:

$k_1 = hf(x,y)$

$k_2 = hf\left[x+\frac{h}{2},\ y+\frac{k_1}{2}\right]$

$k_3 = hf\left[x+\frac{h}{2},\ y+\frac{k_2}{2}\right]$

$k_4 = hf[x+h,\ y+k_3]$

$\Delta y = \frac{1}{6}[k_1+2k_2+2k_3+k_4]$

Now $y_1 = y_0 + \Delta y$.

① Given $\frac{dy}{dx} = x^3+y$, $y(0)=2$. Compute $y(0.2)$, $y(0.4)$ & $y(0.6)$ by RK method of fourth order.

Soln: Gn $y' = x^3 + y$

$x_0 = 0;\ y_0 = 2.\ h = 0.2.$

Now $k_1 = hf(x_0, y_0) = 0.2\,[x_0^3 + y_0] = 0.2(0+2) = 0.4$

$k_2 = hf\left[x_0 + \frac{h}{2},\ y_0 + \frac{k_1}{2}\right] = 0.2\, f\left[0 + \frac{0.2}{2},\ 2 + \frac{0.4}{2}\right]$

$= 0.2\, f[0.1,\ 2.2]$

$= 0.2\,[(0.1)^3 + 2.2] = 0.4402.$

$k_3 = hf\left[x_0 + \frac{h}{2},\ y_0 + \frac{k_2}{2}\right] = 0.2\, f[0.1,\ 2.2201]$

$= 0.2\,[(0.1)^3 + 2.2201] = 0.4442.$

$k_4 = hf[x_0 + h,\ y_0 + k_3] = 0.2\, f[0.2,\ 2.4442]$

$= 0.2\,[(0.1)^3 + 2.4442] = 0.4904$

$\Delta y = \frac{1}{6}[k_1 + 2k_2 + 2k_3 + k_4]$

$= \frac{1}{6}[0.4 + (0.4402)2 + (0.4442)2 + 0.4904]$

$= 0.4432$

$y_1 = y_0 + \Delta y$

$= 2 + 0.4432$

$= 2.4432.$

Now RK method for (x_1, y_1)

$k_1 = h f(x_1, y_1) = 0.4902$.

$k_2 = h f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_1}{2}\right] = 0.5430$

$k_3 = h f\left[x_1 + \frac{h}{2}, y_1 + \frac{k_2}{2}\right] = 0.5483$

$k_4 = h f\left[x_1 + h, y_1 + k_3\right] = 0.6111$

$\Delta y = 0.5473$.

$y_2 = y_1 + \Delta y$

$= 2.4432 + 0.5473$

$= 2.9905$

Now RK method for (x_2, y_2) where $x_2 = 0.4$, $y_2 = 2.9905$

$k_1 = hf(x_2, y_2) = 0.6108$

$k_2 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_1}{2}\right) = 0.6841$

$k_3 = hf\left(x_2 + \frac{h}{2}, y_2 + \frac{k_2}{2}\right) = 0.6914$

$k_4 = hf(x_2 + h, y_2 + k_3) = 0.7795$

$\Delta y = 0.6902.$

$y_3 = y_2 + \Delta y = 2.9905 + 0.6902 = 3.6807$

(2) Using RK method of 4th order solve $y' = \frac{y^2 - x^2}{y^2 + x^2}$ with $y(0) = 1$ at $x = 0.2$.

Soln: 1.1959

(3) Find $y(0.8)$ given that $y' = y - x^2$, $y(0.6) = 1.7379$ by using RK method of 4th order. Take $h = 0.1$.

Soln: $y_1 = y(0.7) = 1.8762$

$y_2 = y(0.8) = 2.0142.$