



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



Taylor's Series:

If $f(z)$ is analytic inside a circle c with centre at $z=a$ then $f(z)$ can be expressed as,

$$f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + \dots + \frac{(z-a)^n}{n!} f^{(n)}(a) + \dots$$

which is convergent at every point inside c . This is called Taylor's Series of $f(z)$ about $z=a$.

Note:

The Taylor's Series of $f(z)$ about the point $z=0$ is given by

$$f(z) = f(0) + \frac{z}{1!} f'(0) + \frac{z^2}{2!} f''(0) + \dots + \frac{z^n}{n!} f^{(n)}(0) + \dots$$

Problems:

1) Expand $f(z) = \log(1+z)$ as Taylor's Series about $z=0$ if $|z| < 1$.

$$\text{Sol: } f(z) = \log(1+z)$$

$$f(0) = \log 1 = 0$$

$$f'(z) = \frac{1}{1+z}$$

$$f'(0) = \frac{1}{1+0} = 1$$

$$f''(z) = \frac{-1}{(1+z)^2}$$

$$f''(0) = \frac{-1}{1} = -1$$

$$f'''(z) = \frac{2}{(1+z)^3}$$

$$f'''(0) = \frac{2}{1} = 2$$

Taylor's series about $z=0$ is given by

$$f(z) = f(0) + \frac{z}{1!} f'(0) + \frac{z^2}{2!} f''(0) + \frac{z^3}{3!} f'''(0) + \dots$$



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$$= 0 + \frac{z}{1!} (1) + \frac{z^2}{2!} (-1) + \frac{z^3}{3!} (2) + \dots$$

$$= z - \frac{z^2}{2} + \frac{z^3}{3} - \dots$$

2) Expand $f(z) = \cos z$ about $z = \frac{\pi}{3}$ in Taylor's series.

sol: $f(z) = \cos z$

$$f\left(\frac{\pi}{3}\right) = \cos \frac{\pi}{3} = \frac{1}{2}$$

$$f'(z) = -\sin z$$

$$f'\left(\frac{\pi}{3}\right) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}$$

$$f''(z) = -\cos z$$

$$f''\left(\frac{\pi}{3}\right) = -\cos \frac{\pi}{3} = -\frac{1}{2}$$

$$f'''(z) = \sin z$$

$$f'''\left(\frac{\pi}{3}\right) = \sin \frac{\pi}{3} = \frac{\sqrt{3}}{2}$$

Taylor's Series is

$$f(z) = f\left(\frac{\pi}{3}\right) + \frac{(z - \frac{\pi}{3})}{1!} f'\left(\frac{\pi}{3}\right) + \frac{(z - \frac{\pi}{3})^2}{2!} f''\left(\frac{\pi}{3}\right) + \dots$$

$$= \frac{1}{2} + \frac{(z - \frac{\pi}{3})}{1!} \left(-\frac{\sqrt{3}}{2}\right) + \frac{(z - \frac{\pi}{3})^2}{2!} \left(-\frac{1}{2}\right) + \frac{(z - \frac{\pi}{3})^3}{3!} \left(\frac{\sqrt{3}}{2}\right) + \dots$$

$$= \frac{1}{2} - \frac{(z - \frac{\pi}{3})}{1!} \left(\frac{\sqrt{3}}{2}\right) - \frac{(z - \frac{\pi}{3})^2}{2!} \left(\frac{1}{2}\right) + \frac{(z - \frac{\pi}{3})^3}{3!} \left(\frac{\sqrt{3}}{2}\right) + \dots$$

3) Expand $f(z) = \sin z$ in a Taylor's Series about $z = \frac{\pi}{4}$.

sol: $f(z) = \sin z$

$$f\left(\frac{\pi}{4}\right) = \sin \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$f'(z) = \cos z$$

$$f'\left(\frac{\pi}{4}\right) = \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}}$$

$$f''(z) = -\sin z$$

$$f''\left(\frac{\pi}{4}\right) = -\sin \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

$$f'''(z) = -\cos z$$

$$f'''\left(\frac{\pi}{4}\right) = -\cos \frac{\pi}{4} = -\frac{1}{\sqrt{2}}$$

Taylor Series formula for $z = a$

$$f(z) = f(a) + \frac{(z-a)}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + \frac{(z-a)^3}{3!} f'''(a) + \dots$$



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$$\text{At } z = \frac{\pi}{4}$$

$$f(z) = f\left(\frac{\pi}{4}\right) + \frac{\left(z - \frac{\pi}{4}\right)}{1!} f'\left(\frac{\pi}{4}\right) + \frac{\left(z - \frac{\pi}{4}\right)^2}{2!} f''\left(\frac{\pi}{4}\right) + \dots$$

$$= \frac{1}{\sqrt{2}} + \left(z - \frac{\pi}{4}\right) \frac{1}{\sqrt{2}} + \frac{\left(z - \frac{\pi}{4}\right)^2}{2} \left(-\frac{1}{\sqrt{2}}\right)$$

$$+ \frac{\left(z - \frac{\pi}{4}\right)^3}{6} \left(-\frac{1}{\sqrt{2}}\right) + \dots$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \left(z - \frac{\pi}{4}\right) - \frac{1}{2\sqrt{2}} \left(z - \frac{\pi}{4}\right)^2 - \frac{1}{6\sqrt{2}} \left(z - \frac{\pi}{4}\right)^3 + \dots$$

$$= \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} \left(z - \frac{\pi}{4}\right) - \frac{1}{2\sqrt{2}} \left(z - \frac{\pi}{4}\right)^2 - \frac{1}{6\sqrt{2}} \left(z - \frac{\pi}{4}\right)^3 + \dots$$

Zeros of an analytic function:

If a function $f(z)$, analytic in a region R , is zero at a point $z = z_0$ in R , then z_0 is called a zero of $f(z)$.

Simple zero:

If $f(z_0) = 0$ and $f'(z_0) \neq 0$, then $z = z_0$ is called a simple zero of $f(z)$ or a zero of the first order.

Zero of order n :

If $f(z_0) = f'(z_0) = \dots = f^{(n-1)}(z_0) = 0$ and $f^{(n)}(z_0) \neq 0$ then z_0 is called zero of order n .

(or)

An analytic function $f(z)$ is said to have a zero of order n if $f(z)$ can be expressed as $f(z) = (z - z_0)^n \phi(z)$, where $\phi(z)$ is analytic and $\phi(z_0) \neq 0$.