



## Unit-III Partial Differential Equations

### Solution of Standard Types of first order Partial differential equations

The partial differential of the first order can be written as  $f(x,y,z,p,q) = 0$ , where  $p = \frac{\partial z}{\partial x}$  and  $q = \frac{\partial z}{\partial y}$ ,  $x, y$  are independent variables and  $z$  is a dependent variable.

#### Standard Type-I

Equations containing  $p$  and  $q$  only.

$$(i) f(p, q) = 0$$

(i) Solve  $\sqrt{p} + \sqrt{q} = 1$

$$\text{Let } \sqrt{p} + \sqrt{q} = 1 \rightarrow (1)$$

This is of type  $f(p, q) = 0$

#### To find the complete integral

Let us assume that

$$z = ax + by + c \rightarrow (2)$$

$$\frac{\partial z}{\partial x} = a, \quad \frac{\partial z}{\partial y} = b$$

$$p = a, \quad q = b$$

Subs the values of  $p, q$  in (1), we get

$$\sqrt{a} + \sqrt{b} = 1$$

$$\sqrt{b} = 1 - \sqrt{a}$$

$$b = (1 - \sqrt{a})^2$$

Subs the value of  $b$  in (2),

$$z = ax + (1 - \sqrt{a})^2 y + c \rightarrow (3)$$

which is the required complete integral.



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(ii) To find the singular Integral.

Difff (3), p.w.r to c, we get

$$\delta = 1 \text{ (which is absurd)}$$

There is no singular integral.

(iii) To find the general Integral.

put  $c = f(a)$  in (3), we get

$$Z = ax + (1-\sqrt{a})^2 y + f(a) \rightarrow (4)$$

$$\frac{\partial Z}{\partial a} = 0 \rightarrow (5)$$

Eliminate a between (4) & (5) we get the general integral.

2. Solve.  $p+q+pq=0$

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$$\text{Given } p+q+pq=0 \rightarrow (1)$$

This is of the form  $f(p, q)=0$

To find the Complete integral

Let us assume  $Z = ax+by+c \rightarrow (2)$

$$\begin{aligned} \frac{\partial Z}{\partial x} &= a & \frac{\partial Z}{\partial y} &= b \\ p &= a & q &= b \end{aligned}$$

Subs the values of p, q in (1),

$$a+b+ab=0$$

$$a+b(1+a)=0$$

$$b(1+a)=-a$$

$$b = -\frac{a}{1+a}$$

Subs the value of b in (2), we get

$Z = ax - \left(\frac{a}{1+a}\right)y + c$  which is the required complete integral.