



Lagrange's linear PDE.

1. Solve $xp + yq = z$.

Soln

Given $xp + yq = z$.

The given PDE is a Lagrange's linear equation with

$$P = x, Q = y, R = z.$$

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}.$$

Taking the 1st two ratios, we get

$$\frac{dx}{x} = \frac{dy}{y}$$

Integrating we get:

$$\log x = \log y + \log c,$$

$$\log x - \log y = \log c,$$

$$\log \left(\frac{x}{y} \right) = \log c,$$

$$\frac{x}{y} = c,$$

$$u = \frac{x}{y}$$



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Taking the last two ratios, we get

$$\frac{dy}{y} = \frac{dz}{z}$$

Integrating we get,

$$\log y = \log z + \log C_2$$

$$\log y - \log z = \log C_2$$

$$\log \left(\frac{y}{z}\right) = \log C_2$$

$$\frac{y}{z} = C_2$$

$$v = \frac{y}{z}$$

The general solution of the given equation $f(u, v) = 0$

$$f\left(\frac{x}{y}, \frac{y}{z}\right) = 0.$$

2. Solve $xp + yq = x$.

Sol: The given PDE is a Lagrange's linear equation with $P = x$, $Q = y$, $R = x$

The subsidiary equation are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{x}$$

Taking the 1st two ratios, we get.

$$\frac{dx}{x} = \frac{dy}{y}$$

Integrating we get.

$$\log x = \log y + \log C_1$$

$$\log x - \log y = \log C_1$$

$$\log \left(\frac{x}{y}\right) = \log C_1$$

$$\frac{x}{y} = C_1$$

$$u = x/y$$



Taking the first and third ratios, we get

$$\frac{dx}{x} = \frac{dz}{z}$$

$$dx = dz$$

Integrating we get.

$$x = z + C_2$$

$$C_2 = x - z$$

$$v = x - z$$

The general solution of the given equation is $f(u, v) = 0$

$$f\left(\frac{x}{y}, x - z\right) = 0$$

3. Solve $P \tan x + Q \tan y = \tan z$.

Sol

$$\text{Given } P \tan x + Q \tan y = \tan z$$

The given PDE is a Lagrange's linear equation with

$$P = \tan x, \quad Q = \tan y, \quad R = \tan z$$

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{\tan x} = \frac{dy}{\tan y} = \frac{dz}{\tan z}$$

Taking the 1st two ratios, we get

$$\frac{dx}{\tan x} = \frac{dy}{\tan y}$$

$$\cot x \, dx = \cot y \, dy$$

Integrating, we get

$$\log(\sin x) = \log(\sin y) + \log C_1$$

$$= \log(\sin y C_1)$$

$$\sin x = \sin y C_1$$

$$\frac{\sin x}{\sin y} = C_1$$



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$$u = \frac{\sin x}{\sin y}$$

Taking the last two ratio, we get -

$$\frac{dy}{\tan y} = \frac{dz}{\tan z}$$

$$\cot y dy = \cot z dz$$

Integrating, we get -

$$\begin{aligned}\log(\sin y) &= \log(\sin z) + \log C_2 \\ &= \log(\sin z C_2)\end{aligned}$$

$$\sin y = \sin z C_2$$

$$\frac{\sin y}{\sin z} = C_2$$

$$v = \frac{\sin y}{\sin z}$$

The general solution of the given eqn is $f(u, v) = 0$

$$\text{i.e., } f\left(\frac{\sin x}{\sin y}, \frac{\sin y}{\sin z}\right) = 0$$



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