



Problems under methods of Multipliers-

1. Solve $(mx - ny)p + (nz - lz)q = ly - mx$

Sol

The given PDE is a Lagrange's linear equation with.

$$P = mx - ny, Q = nz - lz, R = ly - mx.$$

The subsidiary equation are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$

$$\frac{dx}{mx - ny} = \frac{dy}{nz - lz} = \frac{dz}{ly - mx} \quad \text{--- (1)}$$



Using the multipliers (x, y, z) , each of the ratio is ① is equal to.

$$\frac{x dx + y dy + z dz}{x(mz - ny) + y(nx - lz) + z(ly - mx)} = \frac{x dx + y dy + z dz}{xmx - xny + ynx - lyz + yxz - xzm}$$

$$= \frac{x dx + y dy + z dz}{0}$$

$$\therefore x dx + y dy + z dz = 0$$

Integrating, we get.

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{C_1}{2}$$

$$x^2 + y^2 + z^2 = C_1$$

$$u = x^2 + y^2 + z^2$$

Using the multipliers (l, m, n) , each of the ratio is ① is equal to

$$\frac{l dx + m dy + n dz}{l(mz - ny) + m(nx - lz) + n(ly - mx)} = \frac{l dx + m dy + n dz}{lmz - nly + mnx - lmz + nly - mnx}$$

$$= \frac{l dx + m dy + n dz}{0}$$

$$\therefore l dx + m dy + n dz = 0$$

Integrating, we get,

$$lx + my + nz = C_2$$

$$v = lx + my + nz$$

The general solution of the given equation is $f(u, v) = 0$

$$f(x^2 + y^2 + z^2, lx + my + nz) = 0$$



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2. Solve $(3x-4y)\frac{\partial z}{\partial x} + (4x-2z)\frac{\partial z}{\partial y} = 2y-3x$.

Soln:

The given PDE is a Lagrange's linear equation with.

$$P=3x-4y, Q=4x-2z, R=2y-3x$$

The subsidiary equation are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

Using the multipliers (2,3,4) each of the ratios in (1) is equal to.

$$\frac{2dx+3dy+4dz}{2(3x-4y)+3(4x-2z)+4(2y-3x)} = \frac{2dx+3dy+4dz}{6x-8y+12x-6z+8y-12x} = \frac{2dx+3dy+4dz}{0}$$

Integrating, we get.

$$2x+3y+4z=C_1$$

$$u = 2x+3y+4z$$

Using the multipliers (x,y,z) each of the multipliers in (1) is equal to.

$$\frac{xdx+ydy+zdz}{x(3x-4y)+y(4x-2z)+z(2y-3x)} = \frac{xdx+ydy+zdz}{3x^2-4xy+4xy-2yz+2yz-3xz} = \frac{xdx+ydy+zdz}{0}$$

$$\therefore xdx+ydy+zdz=0$$

Integrating, we get.

$$\frac{x^2}{2} + \frac{y^2}{2} + \frac{z^2}{2} = \frac{C_2}{2}$$

$$x^2+y^2+z^2=C_2$$

$$V = x^2+y^2+z^2$$

$$f(u,v)=0 \quad f(2x+3y+4z, x^2+y^2+z^2)=0$$



3. Solve $x(y-z)p + y(z-x)q = z(x-y)$.

Sol

The given PDE is a Lagrange's linear equation with

$$P = x(y-z), \quad Q = y(z-x), \quad R = z(x-y).$$

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

$$\frac{dx}{x(y-z)} = \frac{dy}{y(z-x)} = \frac{dz}{z(x-y)} \quad \text{--- (i)}$$

Using the multipliers (1, 1, 1) each of the ratios in (i) is equal to

$$\frac{dx+dy+dz}{x(y-z)+y(z-x)+z(x-y)} = \frac{dx+dy+dz}{xy-xz+yz-xy+xz-zy} = \frac{dx+dy+dz}{0}$$

$$\therefore dx+dy+dz = 0$$

Integrating we get

$$x+y+z = c_1$$

$$u = x+y+z.$$

Using the multipliers $(\frac{1}{x}, \frac{1}{y}, \frac{1}{z})$, each of the ratios in (i) is equal to

$$\frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{\frac{1}{x}x(y-z) + \frac{1}{y}y(z-x) + \frac{1}{z}z(x-y)} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{y-z+z-x+x-y} = \frac{\frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz}{0}$$

$$\therefore \frac{1}{x}dx + \frac{1}{y}dy + \frac{1}{z}dz = 0$$

Integrating, we get.

$$\log x + \log y + \log z = \log C_2$$

$$\log (xyz) = \log C_2$$

$$xyz = C_2$$

$$v = xyz$$