



Problem on (0, 2l)

1. $f(x) = (l-x)^2, (0, 2l)$

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right]$$

(1)

$$a_0 = \frac{1}{l} \int_0^{2l} f(x) dx$$

$$= \frac{1}{l} \int_0^{2l} (l-x)^2 dx = \frac{1}{l} \left[\frac{(l-x)^3}{-3} \right]_0^{2l}$$

$$= -\frac{1}{3l} [(-l)^3 - l^3] = \frac{2l^3}{3l}$$

$$a_0 = \frac{2}{3} l^2$$

$$a_n = \frac{1}{l} \int_0^{2l} f(x) \cdot \cos \frac{n\pi x}{l} dx$$

$$= \frac{1}{l} \int_0^{2l} (l-x)^2 \cos \frac{n\pi x}{l} dx$$

$$\begin{aligned} u &= (l-x)^2 & v &= \cos \frac{n\pi x}{l} \\ u' &= -2(l-x) & v_1 &= \frac{1}{n\pi} \sin \frac{n\pi x}{l} \\ u'' &= 2 & v_2 &= \frac{-l^2}{n^2\pi^2} \cos \frac{n\pi x}{l} \\ u''' &= 0 & v_3 &= \frac{l^3}{n^3\pi^3} \sin \frac{n\pi x}{l} \end{aligned}$$



$$= \frac{1}{l} \left[(l-x)^2 \frac{l}{n\pi} \sin \frac{n\pi x}{l} + 2(l-x) \frac{l^2}{n^2 \pi^2} \cos \frac{n\pi x}{l} + 2 \left(\frac{l^3}{n^3 \pi^3} \sin \frac{n\pi x}{l} \right) - 0 \right]_0^{2l}$$

$$= \frac{1}{l} \left[+ \frac{2l^3}{n^2 \pi^2} + \frac{2l^3}{n^2 \pi^2} \right] = \frac{1}{l} \left[\frac{4l^3}{n^2 \pi^2} \right]$$

$$a_n = \frac{4l^2}{n^2 \pi^2}$$

$$b_n = \frac{1}{l} \int_0^{2l} f(x) \sin \frac{n\pi x}{l} dx$$

$$= \frac{1}{l} \int_0^{2l} (l-x)^2 \sin \frac{n\pi x}{l} dx \quad \left| \begin{array}{l} \int u v dx = uv - u'v_2 + u''v_3 - \dots \\ u = (l-x)^2 \quad v = \sin \frac{n\pi x}{l} \\ u' = -2(l-x) \quad v_1 = -\frac{l}{n\pi} \cos \frac{n\pi x}{l} \\ u'' = 2 \quad v_2 = \frac{l^2}{n^2 \pi^2} \sin \frac{n\pi x}{l} \\ u''' = 0 \quad v_3 = -\frac{l^3}{n^3 \pi^3} \cos \frac{n\pi x}{l} \end{array} \right.$$

$$= \frac{1}{l} \left[(l-x)^2 \left(-\frac{l}{n\pi} \cos \frac{n\pi x}{l} \right) - 2(l-x) \frac{l^2}{n^2 \pi^2} \sin \frac{n\pi x}{l} + 2 \frac{l^3}{n^3 \pi^3} \cos \frac{n\pi x}{l} \right]_0^{2l}$$

$$= \frac{1}{l} \left[\left(-\frac{l^3}{n\pi} + \frac{2l^3}{n^3 \pi^3} \right) - \left(-\frac{l^3}{n\pi} + \frac{2l^3}{n^3 \pi^3} \right) \right]$$

$$b_n = 0$$

$$\begin{aligned} (1) \Rightarrow f(x) &= \frac{2l^2/3}{2} + \sum_{n=1}^{\infty} \left[\frac{4l^2}{n^2 \pi^2} \cos n\pi x + 0 \right] \\ &= \frac{l^2}{2} + \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n\pi x \end{aligned}$$

Q. Obtain the Fourier series for $f(x) = \begin{cases} l-x, & 0 < x \leq l \\ 0, & l \leq x < 2l \end{cases}$

Soln.:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right] \rightarrow (1)$$



UNIT-IV FOURIER SERIES AND FOURIER TRANSFORM

FOURIER SERIES- PROBLEMS ON (0,2L)

$$\begin{aligned}
 a_0 &= \frac{1}{l} \int_0^{2l} f(x) dx \\
 &= \frac{1}{l} \left[\int_0^l (l-x) dx + 0 \right] = \frac{1}{l} \left[\int_0^l (l-x) dx \right] \\
 &= \frac{1}{l} \left[\frac{(l-x)^2}{-2} \right]_0^l \\
 &= \frac{1}{2l} [0 - l^2]
 \end{aligned}$$

$$a_0 = l/2$$

$$a_n = \frac{1}{l} \int_0^{2l} f(x) \cos \frac{n\pi x}{l} dx$$

$$= \frac{1}{l} \int_0^l (l-x) \cos \frac{n\pi x}{l} dx$$

$$\begin{aligned}
 \int u v dx &= u v_1 - u' v_2 + u'' v_3 - \dots \\
 u &= l-x & v &= \cos \frac{n\pi x}{l} \\
 u' &= -1 & v_1 &= \frac{l \sin \frac{n\pi x}{l}}{n\pi} \\
 u'' &= 0 & v_2 &= \frac{-l^2 \cos \frac{n\pi x}{l}}{n^2 \pi^2}
 \end{aligned}$$

$$= \frac{1}{l} \left[(l-x) \frac{l}{n\pi} \sin \frac{n\pi x}{l} + \left(\frac{l^2}{n^2 \pi^2} \cos \frac{n\pi x}{l} \right) \right]_0^l$$

$$= \frac{1}{l} \left[\frac{-l^2}{n^2 \pi^2} (-1)^n + \frac{l^2}{n^2 \pi^2} \right]$$

$$= \frac{1}{l} \frac{l^2}{n^2 \pi^2} [1 - (-1)^n]$$

$$a_n = \frac{l}{n^2 \pi^2} [1 - (-1)^n]$$

$$b_n = \frac{1}{l} \int_0^{2l} f(x) \sin \frac{n\pi x}{l} dx$$

$$= \frac{1}{l} \int_0^l (l-x) \sin \frac{n\pi x}{l} dx$$

$$\begin{aligned}
 \int u v dx &= u v_1 - u' v_2 + u'' v_3 - \dots \\
 u &= l-x & v &= \sin \frac{n\pi x}{l} \\
 u' &= -1 & v_1 &= \frac{-l \cos \frac{n\pi x}{l}}{n\pi} \\
 u'' &= 0 & v_2 &= \frac{-l^2 \sin \frac{n\pi x}{l}}{n^2 \pi^2}
 \end{aligned}$$



$$= \frac{1}{l} \left[(l-x) \left(\frac{-l}{n\pi} \cos \frac{n\pi x}{l} \right) + \left(\frac{-l^2}{n^2 \pi^2} \sin \frac{n\pi x}{l} \right) \right]_0^l$$

$$= \frac{1}{l} \left[+ \frac{l^2}{n\pi} \right]$$

$$b_n = \frac{l}{n\pi}$$

$$\begin{aligned} \therefore f(x) &= \frac{l/2}{2} + \sum_{n=1}^{\infty} \left[\frac{l}{n^2 \pi^2} \left[1 - (-1)^n \right] \cos \frac{n\pi x}{l} + \frac{l}{n\pi} \sin \frac{n\pi x}{l} \right] \\ &= \frac{l}{4} + \frac{l}{\pi^2} \sum_{n=1}^{\infty} \frac{[1 - (-1)^n]}{n^2} \cos \frac{n\pi x}{l} + \frac{l}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi x}{l} \end{aligned}$$

3]. Find the fourier series for $f(x) = 2x - x^2$, $0 < x < 2$
 $2l = 2$
 $l = 1$

Soln.:

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} \left[a_n \cos \frac{n\pi x}{l} + b_n \sin \frac{n\pi x}{l} \right]$$

$$a_0 = \frac{1}{l} \int_0^{2l} f(x) dx = \int_0^2 (2x - x^2) dx$$

$$= \left(2 \frac{x^2}{2} - \frac{x^3}{3} \right)_0^2$$

$$= \left(4 - \frac{8}{3} \right) - 0$$

$$a_0 = \frac{4}{3}$$

$$a_n = \frac{1}{l} \int_0^{2l} f(x) \cos \frac{n\pi x}{l} dx = \int_0^2 (2x - x^2) \cos n\pi x dx$$

$$\int u v dx = u v_1 - u' v_2 + u'' v_3 - \dots$$

$$u = 2x - x^2$$

$$u' = 2 - 2x$$

$$u'' = -2$$

$$u''' = 0$$

$$v = \cos n\pi x$$

$$v_1 = \frac{\sin(n\pi x)}{n\pi}$$

$$v_2 = -\frac{\cos(n\pi x)}{n^2 \pi^2}$$

$$v_3 = -\frac{\sin(n\pi x)}{n^3 \pi^3}$$



UNIT-IV FOURIER SERIES AND FOURIER TRANSFORM

FOURIER SERIES- PROBLEMS ON (0,2L)

$$\begin{aligned}
 &= \left[(2x - x^2) \frac{\sin(n\pi x)}{n\pi} + (2 - 2x) \frac{\cos(n\pi x)}{n^2\pi^2} - 2 \left(-\frac{\sin(n\pi x)}{n^3\pi^3} \right) \right]_0^{2l} \\
 &= \frac{-2}{n^2\pi^2} - \frac{2}{n^2\pi^2} \\
 a_n &= \frac{-4}{n^2\pi^2} \\
 b_n &= \frac{1}{2} \int_0^{2l} f(x) \sin \frac{n\pi x}{l} dx \\
 &= \int_0^{2l} (2x - x^2) \sin(n\pi x) dx \\
 &= \left[(2x - x^2) \left(-\frac{\cos n\pi x}{n\pi} \right) - (2 - 2x) \left(\frac{\sin n\pi x}{n^2\pi^2} \right) \right. \\
 &\quad \left. - 2 \left(\frac{\cos n\pi x}{n^3\pi^3} \right) + 0 \right]_0^{2l} \\
 &= \frac{-2}{n^3\pi^3} + \frac{2}{n^3\pi^3} \\
 b_n &= 0 \\
 f(x) &= \frac{2}{3} - \frac{4}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{l}
 \end{aligned}$$

$$\begin{aligned}
 u &= 2x - x^2 & v &= \sin n\pi x \\
 u' &= 2 - 2x & v_1 &= -\frac{\cos n\pi x}{n\pi} \\
 u'' &= -2 & v_2 &= -\frac{\sin n\pi x}{n^2\pi^2} \\
 u''' &= 0 & v_3 &= \frac{\cos n\pi x}{n^3\pi^3}
 \end{aligned}$$