



problems on  $(-l, l)$   
 ①  
 II. find the power series of  $f(x) = \begin{cases} l+x, & -l \leq x \leq 0 \\ l-x, & 0 \leq x \leq l \end{cases}$

Soln.:

$$\text{Now, } f(x) = \begin{cases} \phi_1(x), & -l \leq x \leq 0 \\ \phi_2(x), & 0 \leq x \leq l \end{cases}$$

$$\phi_1(x) = l+x; \quad \phi_2(x) = l-x$$

$$\text{Now } \phi_1(-x) = l-x = \phi_2(x)$$

$$\Rightarrow f(x) \text{ is even } \Rightarrow b_n = 0$$

$$a_0 = \frac{2}{l} \int_0^l f(x) dx = \frac{2}{l} \int_0^l (l-x) dx$$

$$= \frac{2}{l} \left[ \frac{(l-x)^2}{-2} \right]_0^l$$

$$= \frac{1}{l} [0 - l^2] = l$$

$$a_0 = l$$

$$a_n = \frac{2}{l} \int_0^l f(x) \cos \frac{n\pi x}{l} dx$$

$$= \frac{2}{l} \int_0^l (l-x) \cos \frac{n\pi x}{l} dx \quad \int u v dx = uv - u'v_2 + \dots$$

$$= \frac{2}{l} \left[ (l-x) \sin \frac{n\pi x}{l} \left( \frac{l}{n\pi} \right) - (-1) \frac{l^2}{n^2 \pi^2} \left( -\cos \frac{n\pi x}{l} \right) \right]_0^l$$

$$= \frac{2}{l} \left[ (l-x) \frac{l}{n\pi} \sin \frac{n\pi x}{l} - \frac{l^2}{n^2 \pi^2} \cos \frac{n\pi x}{l} \right]_0^l$$

$$= \frac{2}{l} \left[ \frac{-l^2}{n^2 \pi^2} (-1)^n + \frac{l^2}{n^2 \pi^2} \right] = \frac{2}{l} \frac{l^2}{n^2 \pi^2} [1 - (-1)^n]$$

$$= \frac{2l}{n^2 \pi^2} [1 - (-1)^n]$$

$$a_n = \begin{cases} 4l/n^2 \pi^2, & \text{if } n \text{ is odd} \\ 0, & \text{if } n \text{ is even} \end{cases}$$



$$\begin{aligned}\therefore f(x) &= \frac{f}{2} + \sum_{n=\text{odd}}^{\infty} \frac{4f}{n^2\pi^2} \cos nx \\ &= \frac{f}{2} + \frac{4f}{\pi^2} \sum_{n=\text{odd}}^{\infty} \frac{1}{n^2} \cos nx\end{aligned}$$