Type - III Standard type :

1. Solve $z = pq$.

Sol

Given $z = pq$ — (1)

The given PDE is of the form $f(z, p, q) = 0$

(i) To find the complete integral

put $u = x + ay$

$$p = \frac{dz}{du} \; ; \quad q = a\frac{dz}{du}.$$

Sub. the values of p and q in ①, we get.

$$\frac{dz}{du}\, a \frac{dz}{du} = z.$$

$$a\left(\frac{dz}{du}\right)^2 = z$$

$$\left(\frac{dz}{du}\right)^2 = \frac{z}{a}$$

$$\frac{dz}{du} = \frac{\sqrt{z}}{\sqrt{a}}$$

$$\frac{dz}{\sqrt{z}} = \frac{du}{\sqrt{a}}$$

$$z^{-1/2} dz = \frac{du}{\sqrt{a}}$$

Integrating, we get.

$$\frac{z^{1/2}}{1/2} = \frac{u}{\sqrt{a}} + b$$

$$2\sqrt{z} = \frac{x+ay}{\sqrt{a}} + b \quad \text{—— ②}$$

(ii) To find the singular integral

Diff ② partially w.r to b, we get.

$0 = 1$ (which is absurb)

There is no singular integral.

(iii) To find the general integral

Put $b = f(a)$ in ②, we get

$$2\sqrt{z} = \frac{x+ay}{\sqrt{a}} + f(a) \quad \text{—— ③}$$

$$\frac{\partial z}{\partial a} = 0 \quad \text{—— ④}$$

Eliminate a btw ③ and ④ we get the general integral

2. Solve $z^2 = 1 + p^2 + q^2$

Sol

Gn $z^2 = 1 + p^2 + q^2$ —— ①

This is of the type $f(z, p, q) = 0$

(i) To find the complete integral

Let $u = x + ay$

$p = \frac{dz}{du}$ $\quad q = a\frac{dz}{du}$.

Substituting the values of p and q in ①, we get.

$$z^2 = 1 + \left(\frac{dz}{du}\right)^2 + \left(a\frac{dz}{du}\right)^2$$

$$z^2 = 1 + \left(\frac{dz}{du}\right)^2 + a^2\left(\frac{dz}{du}\right)^2.$$

$$z^2 - 1 = \left(\frac{dz}{du}\right)^2 (1 + a^2).$$

$$\left(\frac{dz}{du}\right)^2 = \frac{z^2 - 1}{1 + a^2}$$

$$\frac{dz}{du} = \frac{\sqrt{z^2 - 1}}{\sqrt{1 + a^2}}$$

$$\frac{dz}{\sqrt{z^2 - 1}} = \frac{du}{\sqrt{1 + a^2}}.$$

Integrating, we get.

$$\cosh^{-1} z = \frac{u}{\sqrt{1 + a^2}} + b$$

$$\cosh^{-1} z = \frac{x + ay}{\sqrt{1 + a^2}} + b \quad \text{—— ②}$$

which is the required complete integral

Diff ② partially w.r.to b, we get

$0 = 1$ (absurd)

There is no singular integral

(iii) To find the general integral

put $b = f(a)$ in ②

$$\cosh^{-1} z = \frac{x+ay}{\sqrt{1+a^2}} + f(a) \quad \text{——} ③$$

$$\frac{\partial z}{\partial a} = 0. \quad \text{——} ④$$

Eliminate a b/w ③ and ④, we get the general integral

Lagrange's linear PDE.

1. Solve $xp + yq = z$.

Soln

Given $xp + yq = z$.

The given PDE is a lagrange's linear equation with

$P = x$, $Q = y$, $R = z$.

The subsidiary equations are $\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$.

$$\frac{dx}{x} = \frac{dy}{y} = \frac{dz}{z}$$

Taking the 1st two ratios, we get.

$$\frac{dx}{x} = \frac{dy}{y}$$

Integrating we get:

$$\log x = \log y + \log c,$$

$$\log x - \log y = \log c,$$

$$\log\left(\frac{x}{y}\right) = \log c,$$

$$\frac{x}{y} = c,$$

$$u = \frac{x}{y}$$

(iii) To find the general integral

put $b = f(a)$ in (2)

$$\cosh^{-1} z = \frac{x + ay}{\sqrt{1+a^2}} + f(a) \quad \text{--- (3)}$$

$$\frac{\partial z}{\partial a} = 0. \quad \text{--- (4)}$$

Eliminate a b/w (3) and (4), we get the general integral