



Unit - III Partial Differential Equations

Solution of Standard Types of first order Partial differential Equations

The partial differential of the first order can be written as $f(x, y, z, p, q) = 0$, where $p = \frac{\partial z}{\partial x}$ and $q = \frac{\partial z}{\partial y}$, x, y are independent variables and z is a dependent variable.

Standard Type - I

Equations containing p and q only.

(i) $f(p, q) = 0$

1) Solve $\sqrt{p} + \sqrt{q} = 1$

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Ans $\sqrt{p} + \sqrt{q} = 1 \rightarrow \text{①}$

This is of type $f(p, q) = 0$

(i) To find the complete integral

Let us assume that

$$z = ax + by + c \rightarrow \text{②}$$

$$\frac{\partial z}{\partial x} = a, \quad \frac{\partial z}{\partial y} = b$$

$$p = a, \quad q = b$$

Subs the values of p, q in ①, we get

$$\sqrt{a} + \sqrt{b} = 1$$

$$\sqrt{b} = 1 - \sqrt{a}$$

$$b = (1 - \sqrt{a})^2$$

Subs the value of b in ②,

$$z = ax + (1 - \sqrt{a})^2 y + c \rightarrow \text{③}$$

which is the required complete integral.



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(i) To find the Singular Integral.

Diff (3), p.w.r.to c, we get
 $0 = 1$ (which is absurd)

There is no singular integral.

(ii) To find the general Integral.

Put $c = f(a)$ in (3), we get

$$Z = ax + (1 - \sqrt{a})^2 y + f(a) \rightarrow (4)$$

$$\frac{\partial Z}{\partial a} = 0 \rightarrow (5)$$

Eliminate a between (4) & (5) we get the general integral.

2. Solve. $p + q + pq = 0$

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Given $p + q + pq = 0 \rightarrow (1)$

This is of the form $f(p, q) = 0$

To find the Complete integral

Let us assume $Z = ax + by + c \rightarrow (2)$

$$\frac{\partial Z}{\partial x} = a \quad \left| \quad \frac{\partial Z}{\partial y} = b \right.$$
$$p = a \quad \left| \quad q = b \right.$$

Subs the values of p, q in (1),

$$a + b + ab = 0$$

$$a + b(1 + a) = 0$$

$$b(1 + a) = -a$$

$$b = \frac{-a}{1 + a}$$

Subs the value of b in (2), we get

$Z = ax - \left(\frac{a}{1 + a}\right)y + c$ which is the required complete integral.