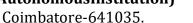


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UNIT 4- COMPLEX INTEGRATION

Taylor's series

Problem 7 Expand $f(z) = \sin z$ into a Taylor's series about $z = \frac{\pi}{4}$. Solution: Given $f(z) = \sin z$ $f'(z) = \cos z$ $f''(z) = -\sin z$ $f^{\prime\prime\prime\prime}(z) = -\cos z$ Here $a = \frac{\pi}{4}$ $\therefore f\left(\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $f'\left(\frac{\pi}{4}\right) = \cos\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$ $f''\left(\frac{\pi}{4}\right) = -\sin\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ $f'''\left(\frac{\pi}{4}\right) = -\cos\left(\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$ W.K.T Taylor's series of f(z) at z = a is $f(z) = f(a) + \frac{z-a}{1!} f'(a) + \frac{(z-a)^2}{2!} f''(a) + \dots$ $f(z) = f\left(\frac{\pi}{4}\right) + \frac{z - \frac{\pi}{4}}{1!} f'\left(\frac{\pi}{4}\right) + \frac{\left(z - \frac{\pi}{4}\right)^2}{2!} f''\left(\frac{\pi}{4}\right) + \dots$

$$=\frac{1}{\sqrt{2}} + \left(z - \frac{\pi}{4}\right) \frac{1}{\sqrt{2}} - \left(\frac{z - \frac{\pi}{4}}{2}\right)^2 \left(\frac{1}{\sqrt{2}}\right) + \dots$$



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Problem 5 Expand $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ as a Taylor series valid in the region |z| < 2. Solution: Given $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ Now $(z+2)(z+3) = z^2 + 5z + 6$ $\therefore \frac{z^2 - 1}{(z+2)(z+3)} = 1 + \frac{-5z - 7}{(z+2)(z+3)}$ Now $\frac{-5z-7}{(z+2)(z+3)} = \frac{A}{z+2} + \frac{B}{z+3}$ -5z-7 = A(z+3) + B(z+2)Put z = -2A = 3Put z = -3B = -8 $\therefore f(z) = 1 + \frac{3}{z+2} - \frac{8}{z+3}$ Given |z| < 2 $f(z) = 1 + \frac{3}{2\left(1 + \frac{z}{2}\right)} - \frac{8}{3\left(1 + \frac{z}{3}\right)}$ $=1+\frac{3}{2}\left(1+\frac{z}{2}\right)^{-1}-\frac{8}{3}\left(1+\frac{z}{3}\right)^{-1}$ $=1+\frac{3}{2}\left(1-\frac{z}{2}+\left(\frac{z}{2}\right)^{2}+...\right)-\frac{8}{3}\left(1-\frac{z}{3}+\left(\frac{z}{3}\right)^{2}+...\right)$

$$= 1 + \frac{3}{2} \sum_{n=0}^{\infty} (-1)^n \left(\frac{z}{2}\right) - \frac{8}{3} \sum_{n=0}^{\infty} (-1)^n \left(\frac{z}{3}\right)$$
$$f(z) = 1 + \sum_{n=0}^{\infty} (-1)^n \left[\frac{3}{2^{n+1}} - \frac{8}{3^{n+1}}\right] z^n .$$