



**Problem 7** Define Removable singularity

**Solution:**

A singular point  $z = z_0$  is called a removable singularity of  $f(z)$  if  $\lim_{z \rightarrow z_0} f(z)$  exists finitely

Example: For  $f(z) = \frac{\sin z}{z}$ ,  $z = 0$  is a removable singularity since  $\lim_{z \rightarrow 0} f(z) = 1$

**Problem 8** Test for singularity of  $\frac{1}{z^2 + 1}$  and hence find corresponding residues.

**Solution:**

$$\text{Let } f(z) = \frac{1}{z^2 + 1} = \frac{1}{(z+i)(z-i)}$$

Here  $z = -i$  is a simple pole

$z = i$  is a simple pole

$$\begin{aligned} \text{Res}(z=i) &= \lim_{z \rightarrow i} (z-i) \frac{1}{(z+i)(z-i)} \\ &= \lim_{z \rightarrow i} \frac{1}{z+i} = \frac{1}{2i} \end{aligned}$$

$$\text{Res}(z=-i) = \lim_{z \rightarrow -i} (z+i) \frac{1}{(z+i)(z-i)} = \frac{1}{-2i}$$



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