



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)



Initial value theorem:

If the Laplace transform of $f(t)$ and $f'(t)$ exists and

$$L[f(t)] = F(s) \text{ then}$$

$$\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s)$$

Proof:

We know that

$$L[f'(t)] = sL[f(t)] - f(0)$$

$$= sF(s) - f(0)$$

$$sF(s) = L[f'(t)] + f(0)$$

$$sF(s) = \int_0^{\infty} e^{-st} f'(t) dt + f(0)$$

Taking limit as $s \rightarrow \infty$ on both sides, we get

$$\lim_{s \rightarrow \infty} sF(s) = \lim_{s \rightarrow \infty} \left\{ \int_0^{\infty} e^{-st} f'(t) dt + f(0) \right\}$$

$$= \lim_{s \rightarrow \infty} \int_0^{\infty} e^{-st} f'(t) dt + f(0)$$

$$= \int_0^{\infty} \lim_{s \rightarrow \infty} e^{-st} f'(t) dt + f(0)$$

$$= 0 + f(0)$$

$$= \lim_{t \rightarrow 0} f(t)$$

$$\text{Hence, } \lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} sF(s)$$



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Final value theorem:

If the Laplace transform of $f(t)$ and $f'(t)$ exists and

$$L[f(t)] = \lim_{s \rightarrow 0} [sF(s)]$$

Proof:

We know that

$$L[f'(t)] = sL[f(t)] - f(0)$$

$$= sF(s) - f(0)$$

$$sF(s) = L[f'(t)] + f(0)$$

Taking limit $s \rightarrow 0$ on both sides, we get

$$\lim_{s \rightarrow 0} [sF(s)] = \lim_{s \rightarrow 0} \left\{ \int_0^{\infty} e^{-st} f'(t) dt + f(0) \right\}$$

$$= \lim_{s \rightarrow 0} \int_0^{\infty} e^{-st} f'(t) dt + f(0)$$

$$= \int_0^{\infty} f'(t) dt + f(0)$$

$$= [f(t)]_0^{\infty} + f(0)$$

$$= f(\infty) - f(0) + f(0)$$

$$= \lim_{t \rightarrow \infty} f(t)$$

$$\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} [sF(s)]$$



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Problems:-

1) Verify the initial and final value theorem for

$$f(t) = 1 + e^{-t}(\sin t + \cos t).$$

Sol:- $F(s) = L[1 + e^{-t}\sin t + e^{-t}\cos t]$

$$= L(1) + L(\sin t)_{s \rightarrow s+1} + L(\cos t)_{s \rightarrow s+1}$$

$$= \frac{1}{s} + \left(\frac{1}{s^2+1}\right)_{s \rightarrow s+1} + \left(\frac{s}{s^2+1}\right)_{s \rightarrow s+1}$$

$$= \frac{1}{s} + \frac{1}{(s+1)^2+1} + \frac{s+1}{(s+1)^2+1}$$

$$= \frac{1}{s} + \frac{s+2}{s^2+2s+2}$$

$$SF(s) = s \left[\frac{1}{s} + \frac{s+2}{s^2+2s+2} \right]$$

Initial value theorem: $\lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} s F(s)$

$$\lim_{t \rightarrow 0} f(t) = \lim_{t \rightarrow 0} [1 + e^{-t}(\sin t + \cos t)] = 1 + 1 = 2$$

$$\lim_{s \rightarrow \infty} SF(s) = \lim_{s \rightarrow \infty} s \left(\frac{1}{s} + \frac{s+2}{s^2+2s+2} \right)$$

$$= \lim_{s \rightarrow \infty} \left(1 + \frac{s^2+2s}{s^2+2s+2} \right) = \lim_{s \rightarrow \infty} \frac{s^2(1+\frac{2}{s})}{s^2(1+\frac{2}{s}+\frac{2}{s^2})}$$

$$= 1 + 1$$

$$= 2$$

$$\text{Hence } \lim_{t \rightarrow 0} f(t) = \lim_{s \rightarrow \infty} SF(s) = 2$$

Initial value theorem is Verified.



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Final Value theorem: $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s)$

$$\lim_{t \rightarrow \infty} f(t) = \lim_{t \rightarrow \infty} \int_0^t 1 + e^{-t}(\sin t + \cos t) dt = 1$$

$$\lim_{s \rightarrow 0} sF(s) = \lim_{s \rightarrow 0} s \left[\frac{1}{s} + \frac{s+2}{s^2+2s+2} \right]$$

$$= 1$$

Hence $\lim_{t \rightarrow \infty} f(t) = \lim_{s \rightarrow 0} sF(s) = 1$

$\therefore$ Final value theorem is verified.

Laplace transform of some Special functions:

Unit Step function:

The unit step function also called Heavisides unit function

is defined as,

$$u(t-a) = \begin{cases} 0, & t < a \\ 1, & t > a \end{cases}$$

This is the unit step functions at $t=a$. It can also be denoted by $H(t-a)$ or $u_a(t)$.

Result:

Laplace Transform of unit step function is $\frac{e^{-as}}{s}$.

ie, $L[u(t-a)] = \frac{e^{-as}}{s}$