$$= \int \frac{1}{2E}\left(\frac{M}{I}\right)^2 dx \iint y^2 \, dA$$

$$= \int \frac{M^2}{2EI^2} dx \; I \quad (\text{where, } I_{xx} = \iint y^2 \, dA)$$

$$\boxed{U_B = \int \frac{M^2}{2EI^2} dx} \qquad \ldots (2.4)$$

The above equation is the general form of strain energy of the structures subjected to bending moment. This equation is also used to solve the rigid-jointed frame structures.

2.1.3 Strain energy in Pure shear loading

Let us consider a section subjected to shearing force Q acting across two of its opposite forces (Figure 2.4). The workdone by Q in a shear displacement of element 'dx' is given by,

$$U = \frac{1}{2} \times Q \times (\text{Shear displacement of element dx})$$

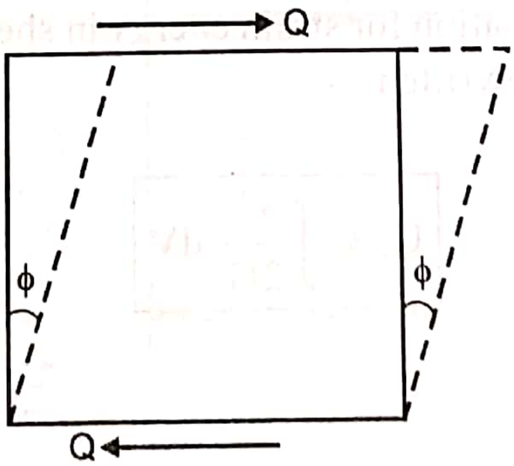

Figure 2.4 Shear loaded structure

$$\text{Shear displacement, dx} = \frac{\tau}{G} x$$

$$dx = \frac{Q}{A_r} \frac{x}{G}$$

Where, A_r → Reduced area

∴ Strain energy, $U = \frac{1}{2} \times Q \times \frac{Qx}{A_r G}$

$$U = \frac{Q^2}{2A_r G} x$$

The total strain energy for any small section is,

$$dU = \frac{Q^2}{2A_r G} dx$$

Integrating above equation,

$$\boxed{U_s = \int \frac{Q^2}{2A_r G} dx} \quad \text{... (2.5)}$$

This is the equation for strain energy in shear loading structure and also it can be rewritten as,

$$\boxed{U_s = \int \frac{\tau^2}{2G} dV} \quad \text{... (2.6)}$$