5

INDUCED STRESSES

Thermal stresses - impact loading - Fatigue - Creep - Stress Relaxation

5.1 THERMAL STRESSES

In a material, a change in temparature can cause a material to change its dimension. The stresses induced in a body due to change in temperature are called thermal stresses. If the temperature increases or decreases, generally the material will expand or contract respectively. This change in a shape is not allowed freely. But if it allowed freely, there will be no stresses acting on it. Ordinarily this change in dimension is linearly related to the temperature variation. If this is the case, the material is homogenous and isotropic, the deformation of a member having a length 'L' can be calculated using the formula.

$$\delta_T = \alpha \, \Delta T \, L \qquad \text{... 5.1}$$

α = Linear coefficient of thermal expansion

ΔT = The algebraic change in temperature of the member

L = The original length of the member

δ_T = The algebraic change in length of the member

If the temperature change varies about its length, then equation 5.1 becomes,

$$\delta_T = \int_0^L \alpha \Delta T \, dx \qquad \text{... 5.2}$$

Then from the above discussion for the body are fixed to rigid supports, the thermal strain can be readily obtained as, the ratio of prevented change in dimension to the original length of the member.

$$\text{i.e. Thermal strain, } \varepsilon_T = \frac{\alpha \Delta T L}{L} = \alpha \Delta T \qquad \text{... 5.3}$$

$$\text{Thermal stress, } \sigma_T = \varepsilon_T E$$

$$\boxed{\sigma_T = \alpha \Delta T E} \qquad \text{... 5.4}$$

When the supports are yield,

$$\text{Actual strain, } \varepsilon = \frac{\alpha \Delta T L - \delta}{L} \qquad \text{... 5.5}$$

$$\text{Actual stress, } \sigma = \left(\frac{\alpha \Delta T L - \delta}{L}\right) \times E \qquad \text{... 5.6}$$

Thermal stress and thermal strain is also known as temperature stress and temperature strain respectively.

> **Note:**
>
> **For a statically determinate structure, the members are free to expand or contract due to temperature variation.**
>
> **For a statically indeterminate structure, the thermal displacements can be constrained by the supports, producing thermal stresses that must by considered in design.**

5.2 IMPACT LOADING

Since, we have dealt only gradually applied loads on a structures. But sometimes, A sudden or dynamic loads applied on a structure. This would be caused by collision of objects. This is called impact loading. In particular, impact occurs when one object strikes another, such that large forces are developed between the object during a very short period of time.

Consider a vertical rod fixed at the upper end and having a collar at the lower end and has a cross-sectional area (A) as shown in figure 5.1. Let the load (P) dropped from a height (h) on the collar. Due to the sudden load (Impact load) the rod gets deformed.

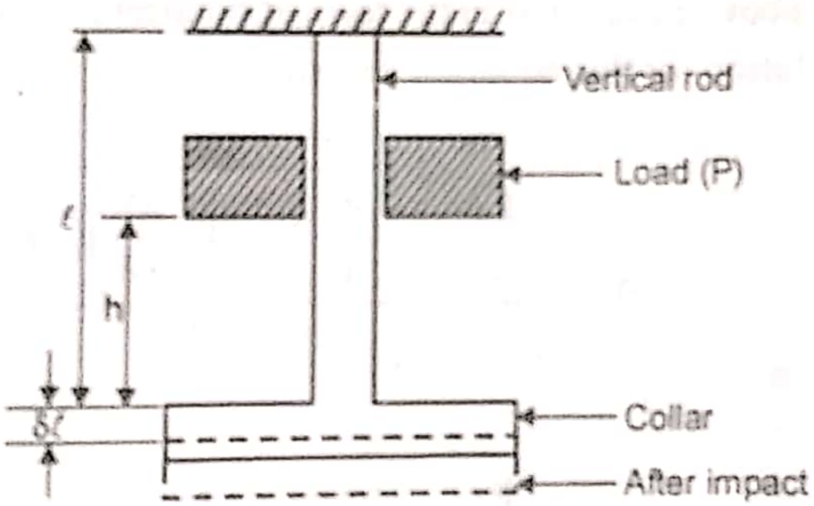

Figure 5.1 Impact Loading

Work done by the Load (W) = Load x distance moved

$$W = P(h + \delta\ell) \qquad \text{... (A)}$$

The strain energy stored by the rod,

$$U = \frac{\sigma^2}{2E} \times V$$

$$U = \frac{\sigma^2}{2E} \times A\ell \qquad \text{... (B)}$$

We know that, $U = W$

From equation A and B, we get

$$\frac{\sigma^2}{2E} \times A\ell = P\,(h + \delta\ell) \qquad \text{... 5.7}$$

$$= P\left(h + \frac{\sigma\ell}{E}\right)$$

$$\frac{\sigma^2 A\ell}{2E} - \frac{P\ell}{E} \cdot \sigma - Ph = 0$$

The above equation is in the form of biharmonic equation. The general solution for this equation becomes,

$$\sigma = \frac{\frac{P\ell}{E} \pm \sqrt{\left(\frac{P\ell}{E}\right)^2 - 4\left(\frac{A\ell}{2E}\right)(-Ph)}}{2(A\ell/2E)}$$

$$= \frac{\frac{P\ell}{E} \pm \sqrt{\left(\frac{P\ell}{E}\right)^2 + \frac{2PA\ell h}{E} \times \left(\frac{P\ell}{E} \times \frac{E}{P\ell}\right)}}{(A\ell/E)}$$

$$= \frac{\frac{P\ell}{E} \pm \frac{P\ell}{E}\sqrt{1 + \frac{2AEh}{P\ell}}}{(A\ell/E)}$$

$$= \frac{\frac{P\ell}{E}\left(1 \pm \sqrt{1 + \frac{2AEh}{P\ell}}\right)}{(A\ell/E)}$$

$$\sigma = \frac{P}{A}\left(1 \pm \sqrt{1 + \frac{2AEh}{P\ell}}\right)$$

For the maximum value of stress, neglect '–ve' roots.

$$\therefore \boxed{\sigma = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2AEh}{P\ell}}\right)} \quad \text{... 5.8}$$

Special cases:

(i) If $\delta\ell <<< h$, from equation 5.7, $\boxed{\because \delta\ell = 0}$

$$\frac{\sigma^2 A\ell}{2E} = P(h + \delta\ell)$$

$$\frac{\sigma^2 A\ell}{2E} = Ph$$

$$\sigma^2 = \frac{2EPh}{A\ell}$$

$$\boxed{\sigma = \sqrt{\frac{2EPh}{A\ell}}} \quad \text{... 5.9}$$

(ii) If h = 0, from equation 5.8.

$$\sigma = \frac{P}{A}\left(1 + \sqrt{1 + 0}\right)$$

$$= \frac{P}{A}(1 + 1)$$

$$\boxed{\sigma = \frac{2P}{A}} \quad \text{... 5.10}$$

The impact factor is obtained using the following expression,

$$n = 1 + \sqrt{1 + \frac{2h}{\delta \ell}} \qquad \text{... 5.11}$$

The impact factor gives the factor responsible for producing the impact load over and above the static load.

5.3 FATIGUE

Some structural members are frequently subjected to cyclic loading over a long period of time. It causes its structures to break down, ultimately leading to fracture. In these circumstances a structural member may failed at a level of stress substantially below the ultimate stress for non-repitive static loads. This phenomena is called fatigue. It is usually responsible for a large percentage of failures in connecting rods, crank shafts, rail road wheels, axels and traffic bridge.

The fatigue cracks are initiated most frequently in a stress concentration areas like holes, notches or sudden changes in structures. In these structural members local stress becomes much greater than average stress acting over the structure. As this higher stress is cycled, it leads to the formation of microscopic cracks. Occurrence of these cracks causes a propagation of cracks. Eventually the cross-sectional area of the member is reduced to point where the load can no longer be sustained, and as a sudden fracture occurs in results. Even though the material known to be ductile, behaves as if it were brittle.

The alternating stress, that can be withstood for a specified number of cycles is called the fatigue strength of the material (Figure 5.2). Some materials possess a stress level that can be withstood for an indefinite number of cycles. This limiting stress is called the endurance or fatigue limit of the material. The results are plotted as a graph representing the stress (S) and number of cycles to failure (N). This graph is called an S–N or Stress–Cycle diagram

(Figure 5.3) and most often the values of N are plotted on a logarithemic scale since they are quitely large.

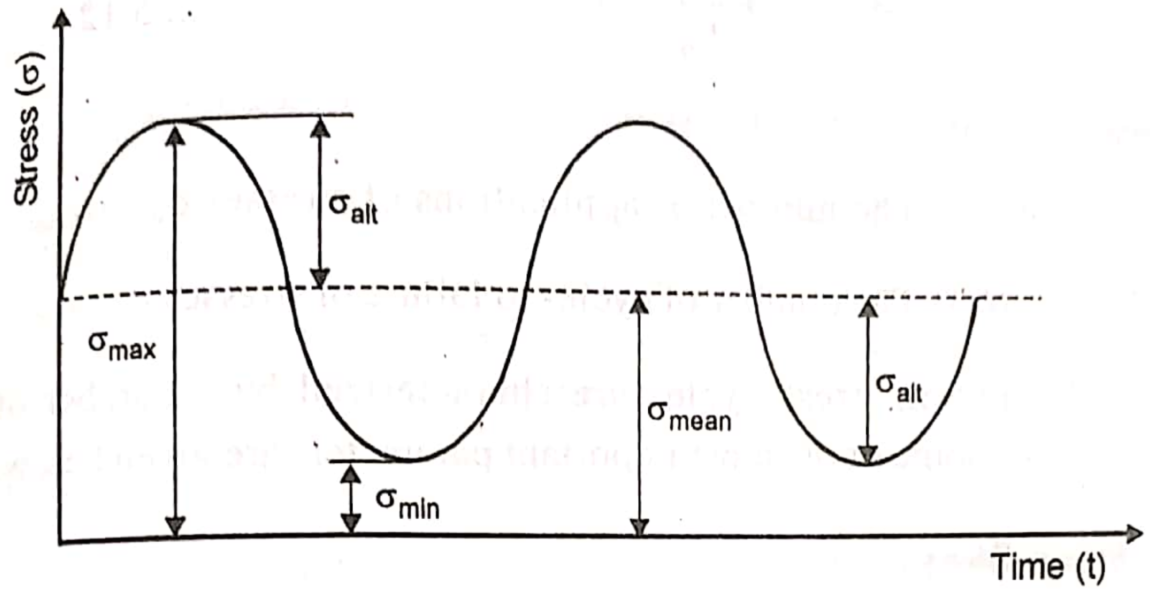

Figure 5.2 Alternating stress in fatigue loading

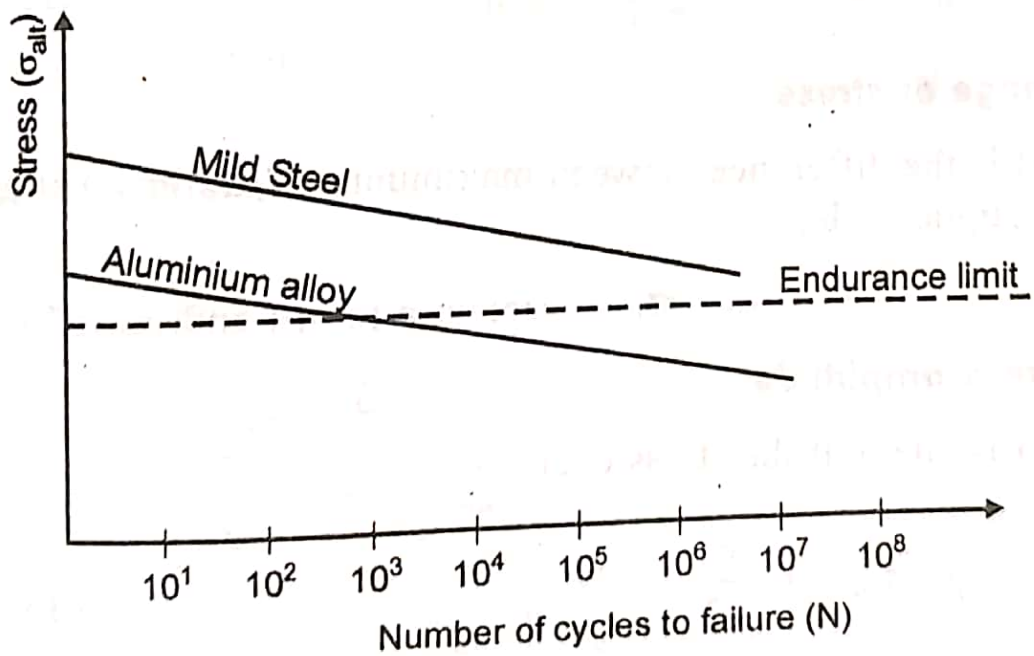

Figure 5.3 S-N curve

In many practical results the amplitude of the alternating stress varies randomly. Therefore, the S-N curve does not apply directly and an alternative means of predicting failure is required. Miner's cumulative damage theory suggests that failure will occur when

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \ldots + \frac{n_r}{N_r} = 1 \qquad \text{... 5.12}$$

where,

$n_1, n_2, \ldots, n_r$ → The number of applications of stresses σ_{alt}, σ_{mean}

$N_1, N_2, \ldots, N_r$ → The number of cycles to failure of stresses σ_{alt}, σ_{mean}

Fluctuating stress cycles are characterized by a number of parameters. Some of the most important parameters are given below:

i) Mean Stress

It is the algebraic sum of maximum and minimum stresses and its average in the fatigue cycle.

$$\sigma_{mean} = \frac{\sigma_{max} + \sigma_{min}}{2} \qquad \text{... 5.13}$$

ii) Range of stress

It is the difference between maximum and minimum stresses in the fatigue cycle.

$$\sigma_r = \sigma_{max} - \sigma_{min} \qquad \text{... 5.14}$$

iii) Stress amplitude

It is one-half the stress cycle.

$$\sigma_{alt} = \frac{\sigma_r}{2} = \frac{\sigma_{max} - \sigma_{min}}{2} \qquad \text{... 5.15}$$

iv) Stress Ratio

It is the ratio between minimum and maximum stresses.

$$R = \frac{\sigma_{min}}{\sigma_{max}} \qquad \text{... 5.16}$$

v) Amplitude Ratio

It is the ratio between alternating stress and mean stress.

$$A = \frac{\sigma_{alt}}{\sigma_{mean}} \quad \text{... 5.17}$$

The fatigue life of a material is generally categorized in the following methods:

i) High-cycle fatigue

From the values of S-N curves, the failure occurs more than 1000 cycles are called high-cycle fatigue.

ii) Low-cycle fatigue

From the values obtained on stress-cycles relations, the failure occurs less than 1000 cycles are called Low-cycle fatigue.

5.3.1 Fatigue strength formulations

Fatigue strength experiments have been carried out over a wide range of stress variations in both tensile and compressive and a typical plot is shown in figure 5.4.

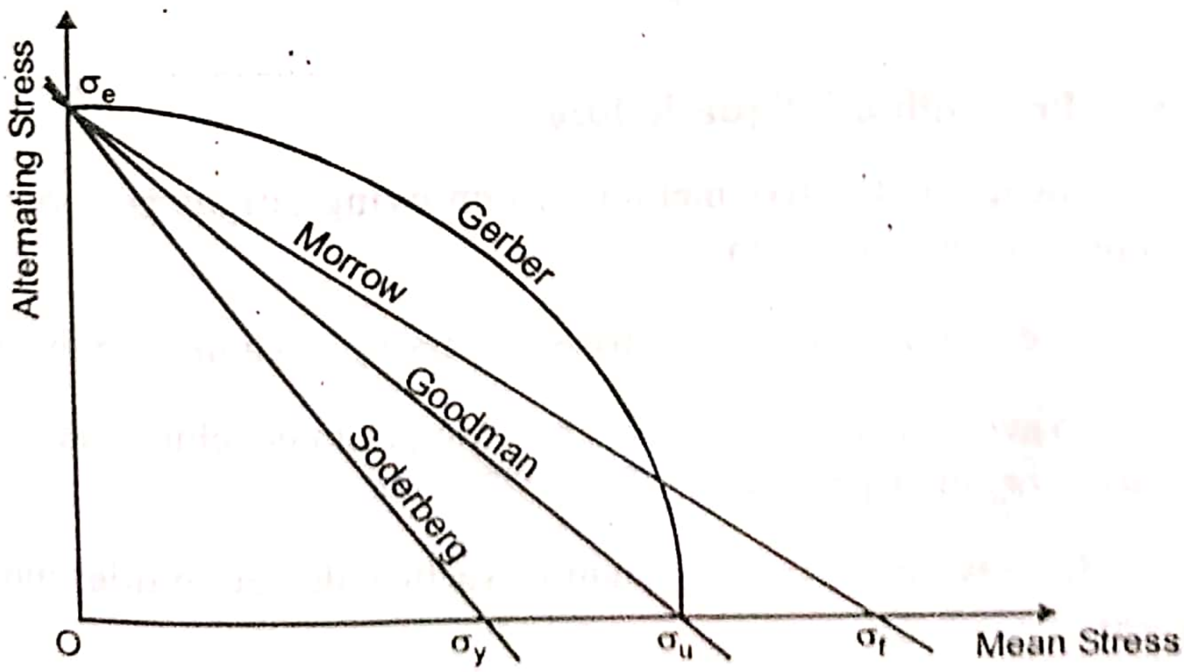

Figure 5.4 Experimental results for fatigue strength

Soderberg proposed a linear variation based on tensile yield strength (σ_y) and is given by,

$$\left(\frac{\sigma_{mean}}{\sigma_y}\right) + \left(\frac{\sigma_{alt}}{\sigma_e}\right) = 1 \quad \text{... 5.18}$$

Goodman approximated a linear variation and is given by,

$$\left(\frac{\sigma_{mean}}{\sigma_u}\right) + \left(\frac{\sigma_{alt}}{\sigma_e}\right) = 1 \quad \text{... 5.19}$$

Gerber proposed a parabolic correlation with ultimate stress (σ_u) and is given by,

$$\left(\frac{\sigma_{mean}}{\sigma_u}\right)^2 + \left(\frac{\sigma_{alt}}{\sigma_e}\right) = 1 \quad \text{... 5.20}$$

Morrow proposed a linear variation based on the true fracture stress (σ_f) and is given by,

$$\left(\frac{\sigma_{mean}}{\sigma_f}\right) + \left(\frac{\sigma_{alt}}{\sigma_e}\right) = 1 \quad \text{... 5.21}$$

5.3.2 Preventing fatigue failure

The most effective method of improving fatigue performance is improvements in design:

To eliminate or reduce stress raisers by streamlining the part.

To avoid sharp surface tears resulting from punching, stamping, shearing or other processes.

To prevent the development of surface discontinuities during processing.