5.2 IMPACT LOADING

Since, we have dealt only gradually applied loads on a structures. But sometimes, A sudden or dynamic loads applied on a structure. This would be caused by collision of objects. This is called impact loading. In particular, impact occurs when one object strikes another, such that large forces are developed between the object during a very short period of time.

Consider a vertical rod fixed at the upper end and having a collar at the lower end and has a cross-sectional area (A) as shown in figure 5.1. Let the load (P) dropped from a height (h) on the collar. Due to the sudden load (Impact load) the rod gets deformed.

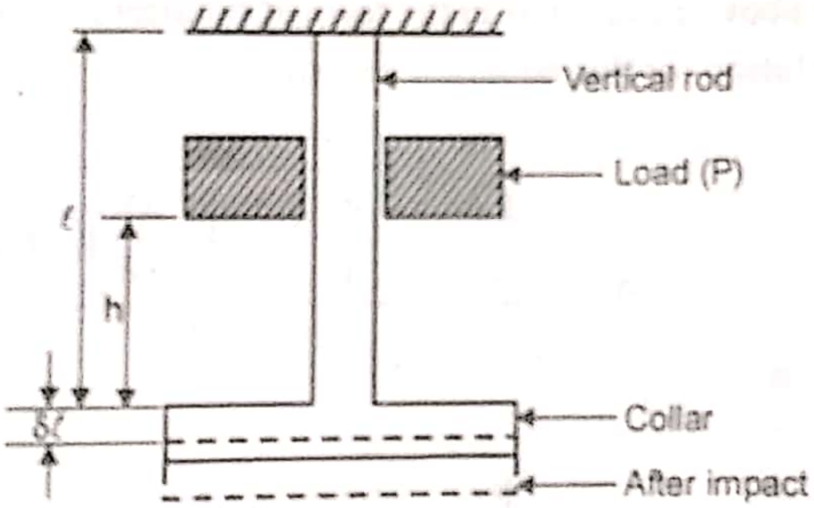

Figure 5.1 Impact Loading

Work done by the Load (W) = Load x distance moved

$$W = P(h + \delta\ell) \qquad \text{... (A)}$$

The strain energy stored by the rod,

$$U = \frac{\sigma^2}{2E} \times V$$

$$U = \frac{\sigma^2}{2E} \times A\ell \qquad \text{... (B)}$$

We know that, $U = W$

From equation A and B, we get

$$\frac{\sigma^2}{2E} \times A\ell = P\,(h + \delta\ell) \qquad \text{... 5.7}$$

$$= P\left(h + \frac{\sigma\ell}{E}\right)$$

$$\frac{\sigma^2 A\ell}{2E} - \frac{P\ell}{E}\cdot\sigma - Ph = 0$$

The above equation is in the form of biharmonic equation. The general solution for this equation becomes,

$$\sigma = \frac{\frac{P\ell}{E} \pm \sqrt{\left(\frac{P\ell}{E}\right)^2 - 4\left(\frac{A\ell}{2E}\right)(-Ph)}}{2(A\ell/2E)}$$

$$= \frac{\frac{P\ell}{E} \pm \sqrt{\left(\frac{P\ell}{E}\right)^2 + \frac{2PA\ell h}{E} \times \left(\frac{P\ell}{E} \times \frac{E}{P\ell}\right)}}{(A\ell/E)}$$

$$= \frac{\frac{P\ell}{E} \pm \frac{P\ell}{E}\sqrt{1 + \frac{2AEh}{P\ell}}}{(A\ell/E)}$$

$$= \frac{\frac{P\ell}{E}\left(1 \pm \sqrt{1 + \frac{2AEh}{P\ell}}\right)}{(A\ell/E)}$$

$$\sigma = \frac{P}{A}\left(1 \pm \sqrt{1 + \frac{2AEh}{P\ell}}\right)$$

For the maximum value of stress, neglect '–ve' roots.

$$\therefore \boxed{\sigma = \frac{P}{A}\left(1 + \sqrt{1 + \frac{2AEh}{P\ell}}\right)} \quad \text{... 5.8}$$

Special cases:

(i) If $\delta\ell <<< h$, from equation 5.7, $\boxed{\because \delta\ell = 0}$

$$\frac{\sigma^2 A\ell}{2E} = P(h + \delta\ell)$$

$$\frac{\sigma^2 A\ell}{2E} = Ph$$

$$\sigma^2 = \frac{2EPh}{A\ell}$$

$$\boxed{\sigma = \sqrt{\frac{2EPh}{A\ell}}} \quad \text{... 5.9}$$

(ii) If h = 0, from equation 5.8.

$$\sigma = \frac{P}{A}\left(1 + \sqrt{1 + 0}\right)$$

$$= \frac{P}{A}(1 + 1)$$

$$\boxed{\sigma = \frac{2P}{A}} \quad \text{... 5.10}$$

The impact factor is obtained using the following expression,

$$n = 1 + \sqrt{1 + \frac{2h}{\delta \ell}} \quad \text{... 5.11}$$

The impact factor gives the factor responsible for producing the impact load over and above the static load.

5.3 FATIGUE

Some structural members are frequently subjected to cyclic loading over a long period of time. It causes its structures to break down, ultimately leading to fracture. In these circumstances a structural member may failed at a level of stress substantially below the ultimate stress for non-repitive static loads. This phenomena is called fatigue. It is usually responsible for a large percentage of failures in connecting rods, crank shafts, rail road wheels, axels and traffic bridge.

The fatigue cracks are initiated most frequently in a stress concentration areas like holes, notches or sudden changes in structures. In these structural members local stress becomes much greater than average stress acting over the structure. As this higher stress is cycled, it leads to the formation of microscopic cracks. Occurrence of these cracks causes a propagation of cracks. Eventually the cross-sectional area of the member is reduced to point where the load can no longer be sustained, and as a sudden fracture occurs in results. Even though the material known to be ductile, behaves as if it were brittle.

The alternating stress, that can be withstood for a specified number of cycles is called the fatigue strength of the material (Figure 5.2). Some materials possess a stress level that can be withstood for an indefinite number of cycles. This limiting stress is called the endurance or fatigue limit of the material. The results are plotted as a graph representing the stress (S) and number of cycles to failure (N). This graph is called an S–N or Stress–Cycle diagram