SNS COLLEGE OF TECHNOLOGY

Coimbatore-35

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DEPARTMENT OF AEROSPACE ENGINEERING

23AST205 Aerospace Structures

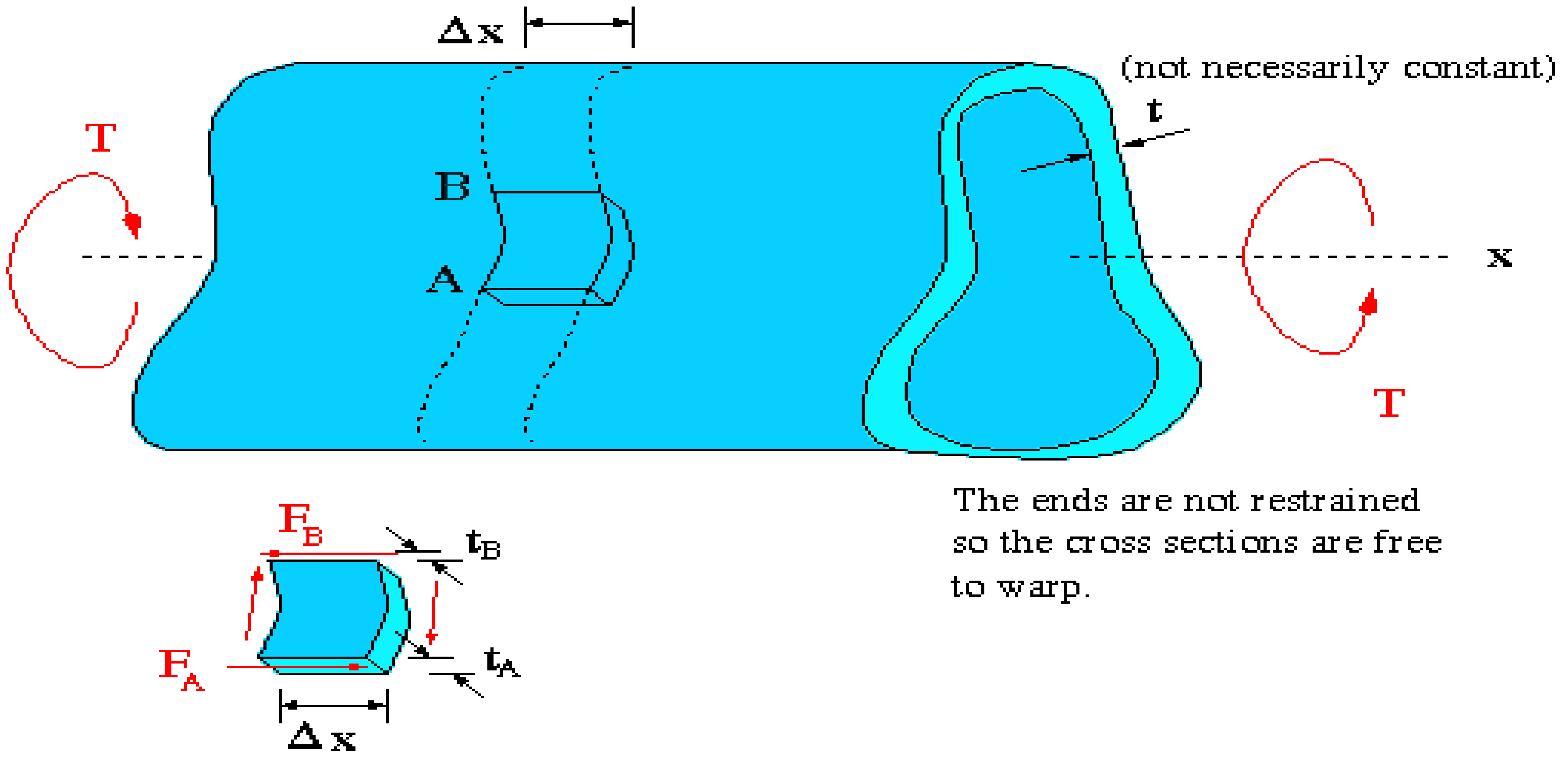
II YEAR III SEM

TORSION OF THIN - WALL CLOSED SECTIONS

Torsion of Thin - Wall Closed Sections

- Derivation

Consider a thin-walled member with a closed cross section subjected to pure torsion.



Examining the equilibrium of a small cutout of the skin reveals that

$$\sum F_x = 0 \quad \Rightarrow \quad F_A - F_B = 0$$

Writing F_A and F_B in terms of the shearing stress at A and B yields

$$\tau_A (t_A \Delta x) - \tau_B (t_B \Delta x) = 0$$

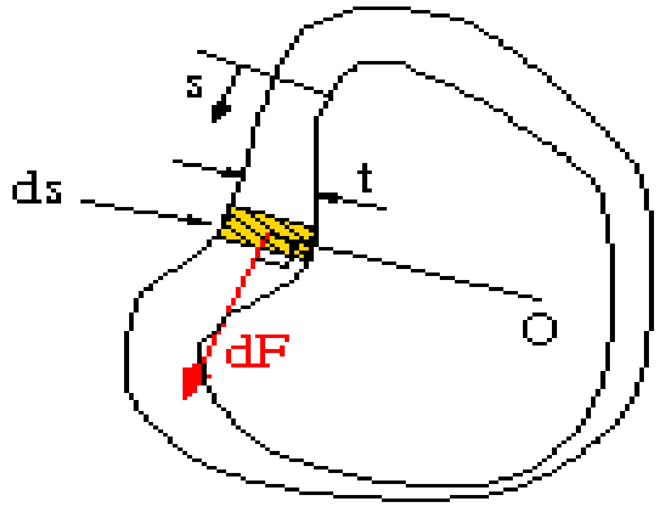
$$\tau_A t_A = \tau_B t_b = \tau t \quad \textit{constant throughout the member}$$

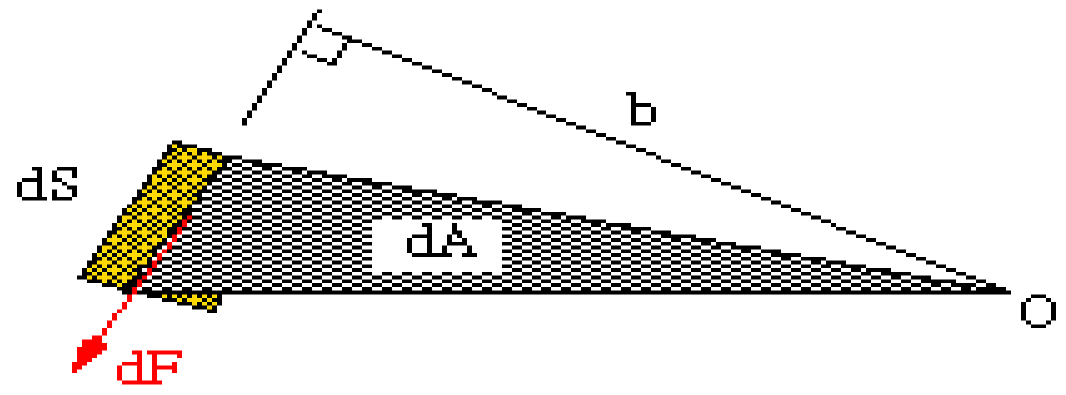
Let τt = q = constant
This is just like the product of VA (velocity x cross sectional area) is constant in a Venturi tube. 'q' is therefore called **shear flow**.

$$dM_O = b\,dF = b\,(q\,ds) = q\,(b\,ds)$$
$$dA = \tfrac{1}{2}\,b\,ds \rightarrow b\,ds = 2\,dA$$

now $dM = q\,(2dA)$

$$T = \oint dM_O = \oint q(2dA)$$





Since q is a constant we have

$$T = 2qA$$

Where A is the area bounded by the centerline of the wall cross section

$$\tau = \frac{T}{2At}$$

Where t is the thickness of the skin at the point considered in the shear stress calculation

ANGLE OF TWIST

By applying strain energy equation due to shear and Castigliano's Theorem the angle of twist for a thin-walled closed section can be shown to be

$$\frac{\delta U}{\delta T} = \frac{\phi}{L} = \frac{T}{4A^2G}\oint\frac{ds}{t}$$

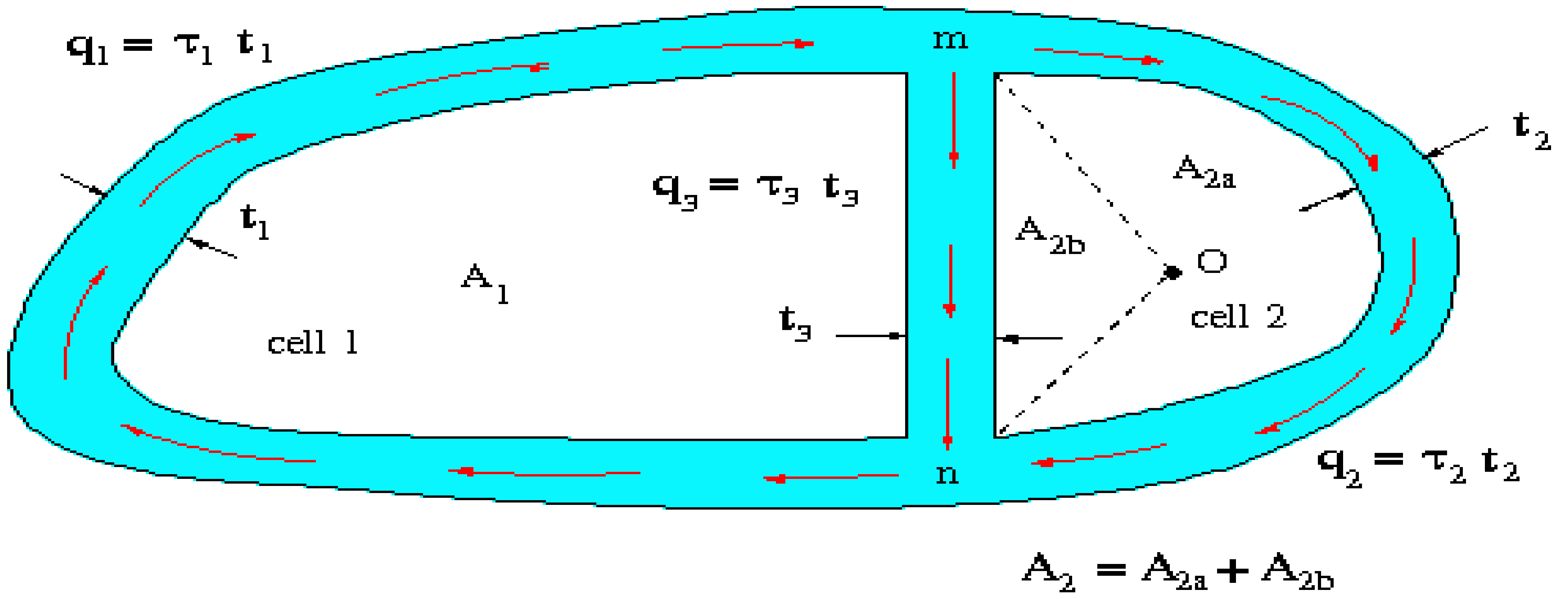
Since T = 2qA, we have

$$\phi = \frac{qL}{2AG}\oint\frac{ds}{t}$$

If the wall thickness is constant along each segment of the cross section, the integral can be replaced by a simple summation

$$\phi = \frac{qL}{2AG}\sum\frac{l}{t}$$

TORSION - SHEAR FLOW RELATIONS IN MULTIPLE-CELL THIN- WALL CLOSED SECTIONS



- The torsional moment in terms of the internal shear flow is simply

$$T_o = 2q_1A_1 + 2q_2A_2 + \ldots\ldots + 2q_nA_n$$

DERIVATION

For equilibrium to be maintained at a exterior-interior wall (or web) junction point (point m in the figure) the shear flows entering should be equal to those leaving the junction

Summing the moments about an arbitrary point O, and assuming clockwise direction to be positive, we obtain

$$q_1 = q_2 + q_3$$

$$\sum M_o = T_O = 2q_1(A_1 + A_{2b}) + 2q_2(A_{2a}) - 2q_3(A_{2b})$$

$$T_O = 2q_1A_1 + 2q_1A_{2b} + 2q_2A_{2a} - 2q_3A_{2b}$$

$$= 2q_1A_1 + 2q_1A_{2b} + 2q_2A_{2a} - 2(q_1 - q_2)A_{2b}$$

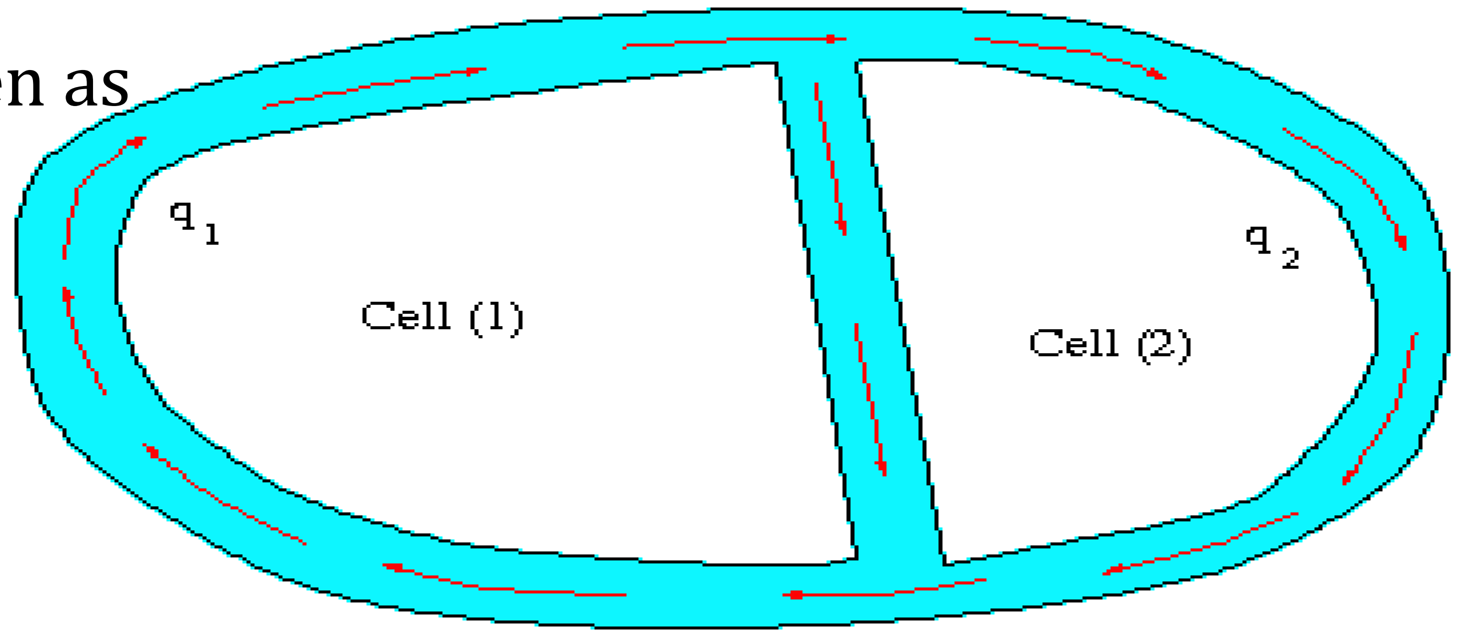
The moment equation above can be simplified to

$$A_2 = A_{2a} + A_{2b}$$

$$T_o = 2q_1A_1 + 2q_2A_2$$

Shear Stress Distribution and Angle of Twist for Two-Cell Thin-Walled Closed Sections

- The equation relating the shear flow along the exterior wall of each cell to the resultant torque at the section is given as



$$T_o = 2q_1A_1 + 2q_2A_2$$

This is a statically indeterminate problem. In order to find the shear flows q1 and q2, the compatibility relation between the angle of twist in cells 1 and 2 must be used. The compatibility requirement can be stated as

$$\phi_1 = \phi_2 = \phi$$

where

$$\phi_1 = \frac{L}{2A_1G}\oint_{cell1}\frac{q}{t}ds$$

$$\phi_2 = \frac{L}{2A_2G}\oint_{cell2}\frac{q}{t}ds$$

$$q_1 = \frac{1}{2}\left[\frac{a_{20} A_1 + a_{12} A}{a_{20} A_1^2 + a_{12} A^2 + a_{10} A_2^2}\right] T \qquad q_2 = \frac{1}{2}\left[\frac{a_{10} A_2 + a_{12} A}{a_{20} A_1^2 + a_{12} A^2 + a_{10} A_2^2}\right] T$$

$$A = A_1 + A_2$$

$$a_{10} = \int \frac{ds}{t} \quad \text{(along exterior wall of cell 1)}$$

$$a_{20} = \int \frac{ds}{t} \quad \text{(along exterior wall of cell 2)}$$

$$a_{12} = \int \frac{ds}{t} \quad \text{(along interior wall between cells 1 \& 2)}$$

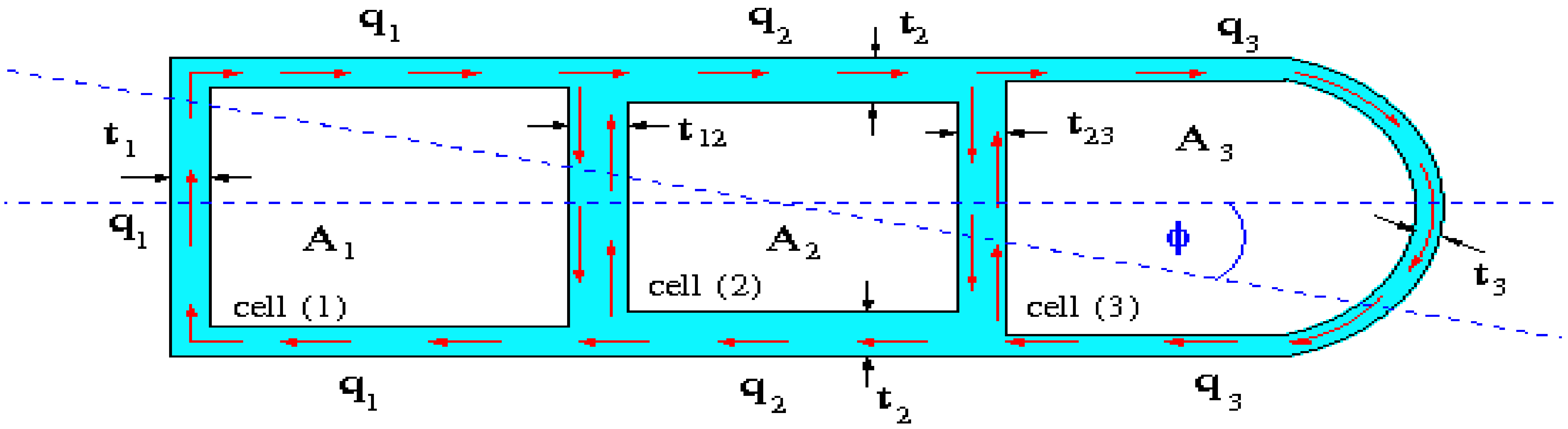
- The shear stress at a point of interest is found according to the equation

$$\tau = \frac{q}{t}$$

- To find the angle of twist, we could use either of the two twist formulas given above. It is also possible to express the angle of twist equation similar to that for a circular section

$$\phi = \frac{TL}{JG} \qquad J = 4\left[\frac{a_{20} A_1^2 + a_{12} A^2 + a_{10} A_2^2}{a_{10} a_{12} + a_{12} a_{20} + a_{10} a_{20}}\right]$$

Shear Stress Distribution and Angle of Twist for Multiple-Cell Thin-Wall Closed Sections



- In the figure above the area outside of the cross section will be designated as **cell (0)**. Thus to designate the exterior walls of cell (1), we use the notation **1-0**. Similarly for cell (2) we use **2-0** and for cell (3) we use **3-0**. The interior walls will be designated by the names of adjacent cells.
- the torque of this multi-cell member can be related to the shear flows in exterior walls as follows

- the torque of this multi-cell member can be related to the shear flows in exterior walls as follows

$$q_1 = \tau_1 t_1 \quad , \quad q_2 = \tau_2 t_2 \quad , \quad q_3 = \tau_3 t_3$$

$$@\ wall\ 1-2 \qquad q_{12} = q_1 - q_2$$

$$@\ wall\ 2-3 \qquad q_{23} = q_2 - q_3$$

$$T = 2q_1A_1 + 2q_2A_2 + 2q_3A_3$$

- For elastic continuity, the angles of twist in all cells must be equal

$$\frac{\phi_1}{L} = \frac{\phi_2}{L} = \frac{\phi_3}{L} = \frac{q}{2AG} \oint \frac{ds}{t} = \frac{\phi}{L}$$

$$Let\ a = \oint \frac{ds}{t}$$

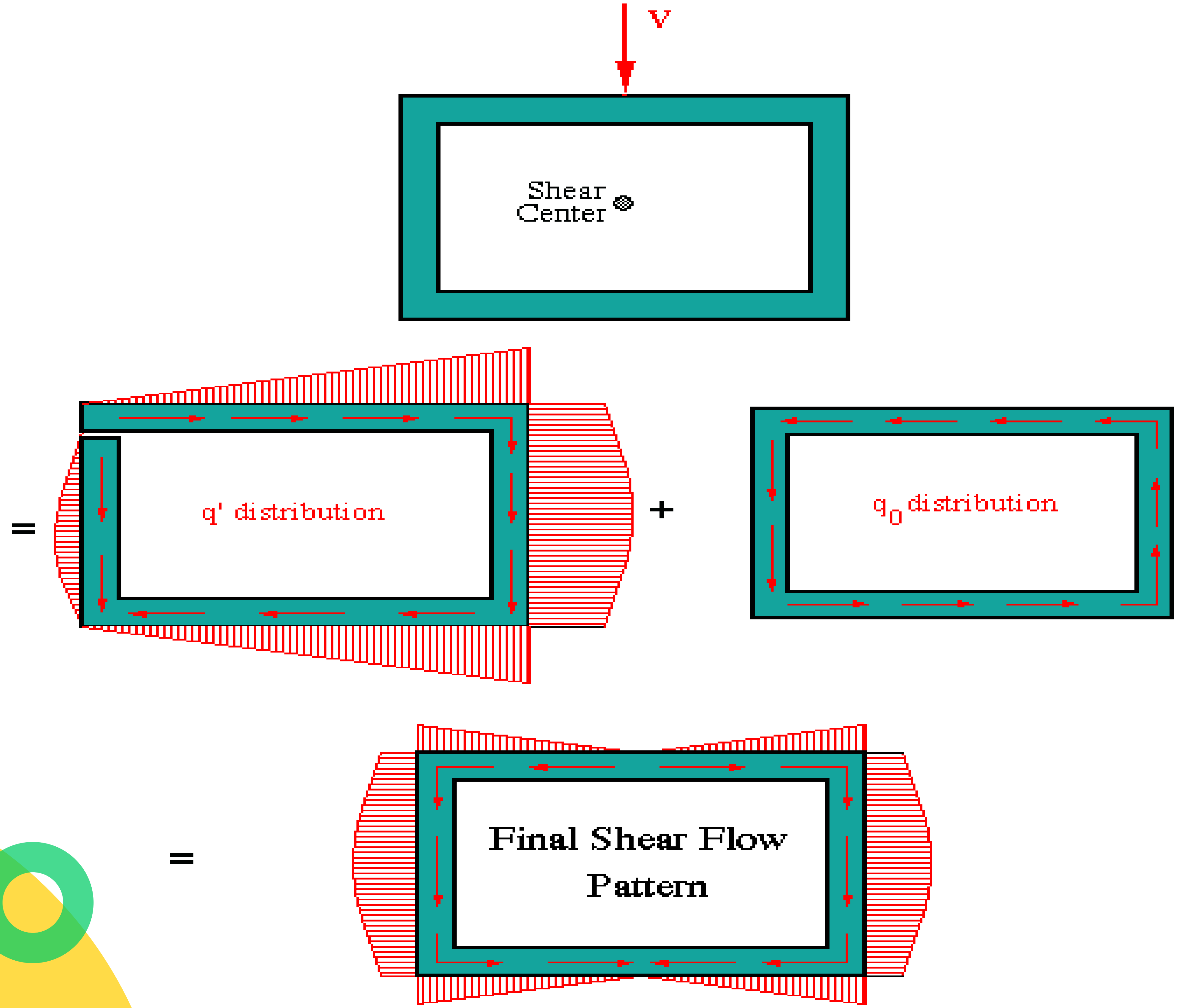
- The direction of twist chosen to be positive is clockwise.

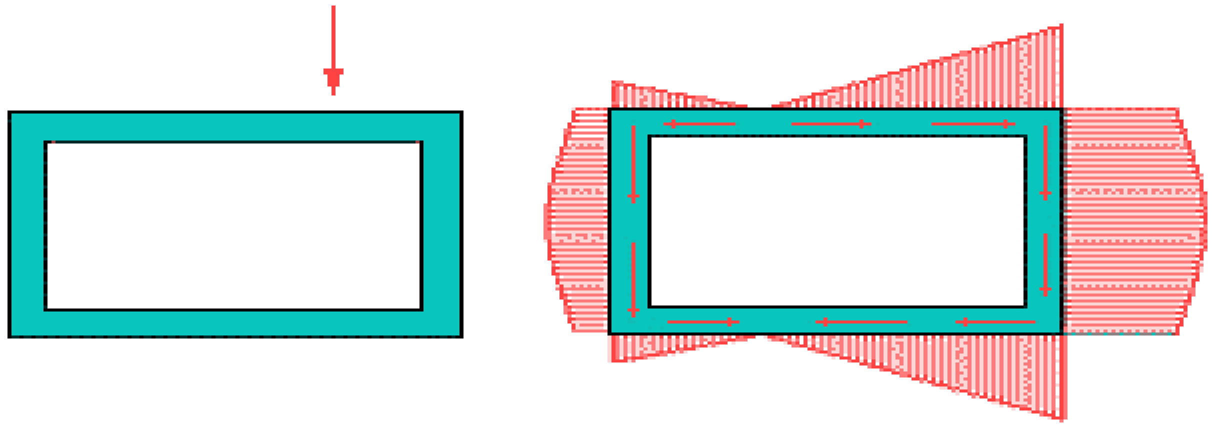
$$\frac{2G\phi}{L} = \frac{1}{A_1}\left[q_1 a_{10} + (q_1 - q_2)a_{12}\right]$$

$$\frac{2G\phi}{L} = \frac{1}{A_2}\left[(q_2 - q_1)a_{12} + q_2 a_{20} + (q_2 - q_3)a_{23}\right]$$

$$\frac{2G\phi}{L} = \frac{1}{A_3}\left[(q_3 - q_2)a_{23} + q_3 a_{30}\right]$$

TRANSVERSE SHEAR LOADING OF BEAMS WITH CLOSED CROSS SECTIONS





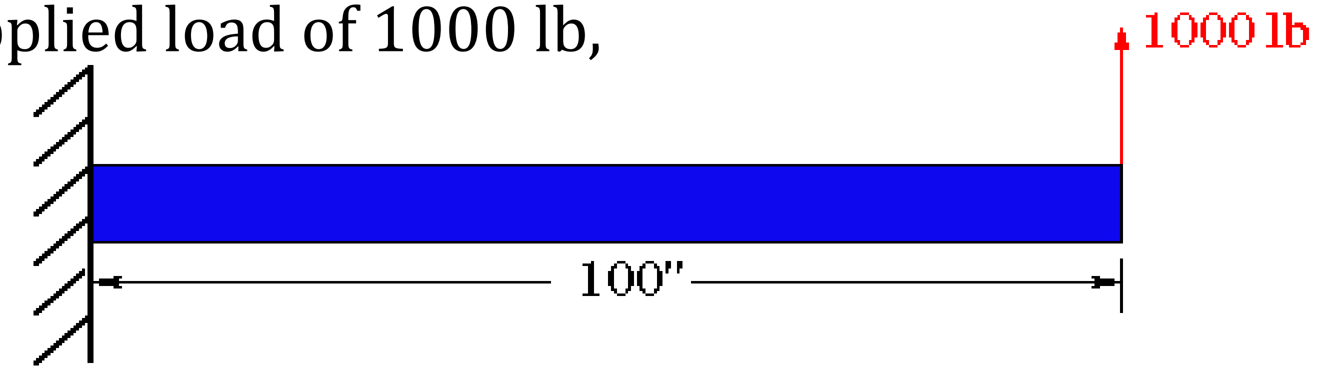
EXAMPLE

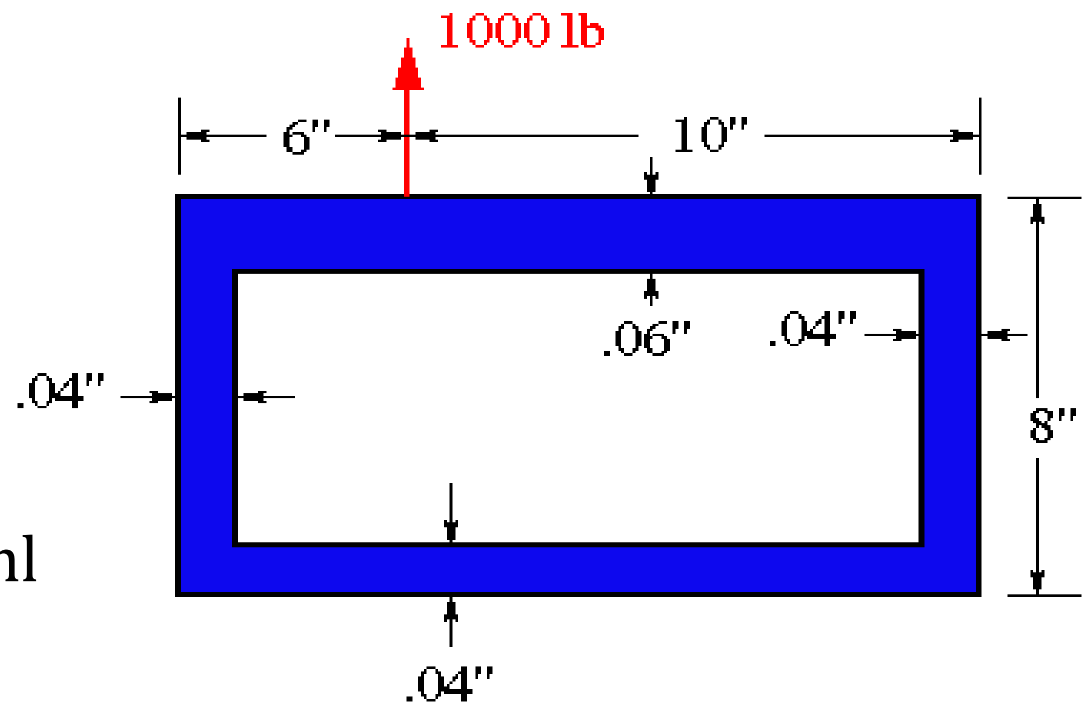
For the thin-walled single-cell rectangular beam and loading shown, determine

(a) the shear center location (ex and ey),

(b) the resisting shear flow distribution at the root section due to the applied load of 1000 lb,

(c) the location and magnitude of the maximum shear stress





ANSWER REFER

http://www.ae.msstate.edu/~masoud/Teaching/exp/A15.2_ex1.html

S.NO	QUESTION	ANSWER
1	Open section	
2	Closed section	
3	Shear flow vs shear center