SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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DEPARTMENT OF AEROSPACE ENGINEERING

23AST206 – AERODYNAMICS

II YEAR IV SEM

UNIT 2 – INCOMPRESSIBLE AND INVISCID FLOWS

TOPIC – VELOCITY POTENTIAL AND STREAM FUNCTION

VELOCITY POTENTIAL AND STREAM FUNCTION

Stream Lines

- Consider 2D incompressible flow
- Continuity Eqn

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) + \frac{\partial}{\partial z}(\rho V_z) = 0$$

$$\frac{\partial}{\partial x}(V_x) + \frac{\partial}{\partial y}(V_y) = 0 \qquad V_y = \int \left(-\frac{\partial V_x}{\partial x} \right) dy$$

VELOCITY POTENTIAL AND STREAM FUNCTION

Stream Function

- What does Stream Function ψ mean?

- Equation for streamlines in 2D are given by

  ψ = constant

- Streamlines may exist in 3D also, but stream function does not
  - Why? (When we work with velocity potential, we may get a perspective)
  - In 3D, streamlines follow the equation

$$\frac{dx}{V_x} = \frac{dy}{V_y} = \frac{dz}{V_z}$$

VELOCITY POTENTIAL AND STREAM FUNCTION

Stream Function- Physical meaning

- Statement: In 2D (viscous or inviscid) flow (incompressible flow OR steady state compressible flow), ψ = constant represents the streamline.
- Proof
- If ψ = constant, then dψ=0

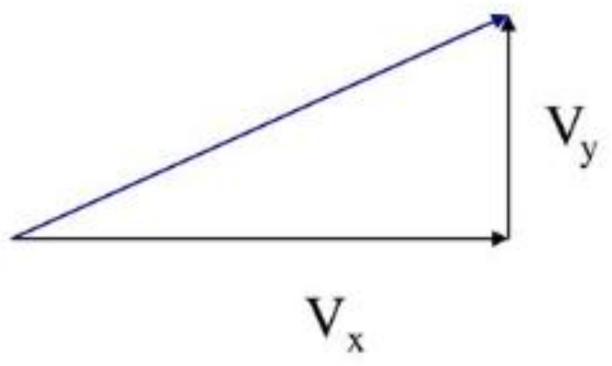
$$d\psi = \left(\frac{\partial \psi}{\partial x}\right)dx + \left(\frac{\partial \psi}{\partial y}\right)dy$$

$$= (-V_y)\,dx + (V_x)\,dy$$

$$= 0$$

- If ψ = constant, then

$$\frac{dy}{dx} = \frac{V_y}{V_x}$$



VELOCITY POTENTIAL AND STREAM FUNCTION

Example 1: A flowrate is described by the equation $\psi = y - x^2$:

1- sketch the streamlines for $\psi=0$, $\psi = 1$ and $\psi = 2$

2- Deriven an expression equation for the velocity V at any point in the flowfield >

3- Calculate the vorticity of this flow.

Solution:

1- for $\psi=0$, $\psi = 1$ and $\psi = 2$

$$2\text{-}\ u = \frac{\partial \psi}{\partial y} = \frac{\partial}{\partial y}(y - x^2) = 1 - 0 = 1$$

$$v = -\frac{\partial \psi}{\partial x} = -\frac{\partial}{\partial x}(y - x^2) = 0 + 2x = 2x$$

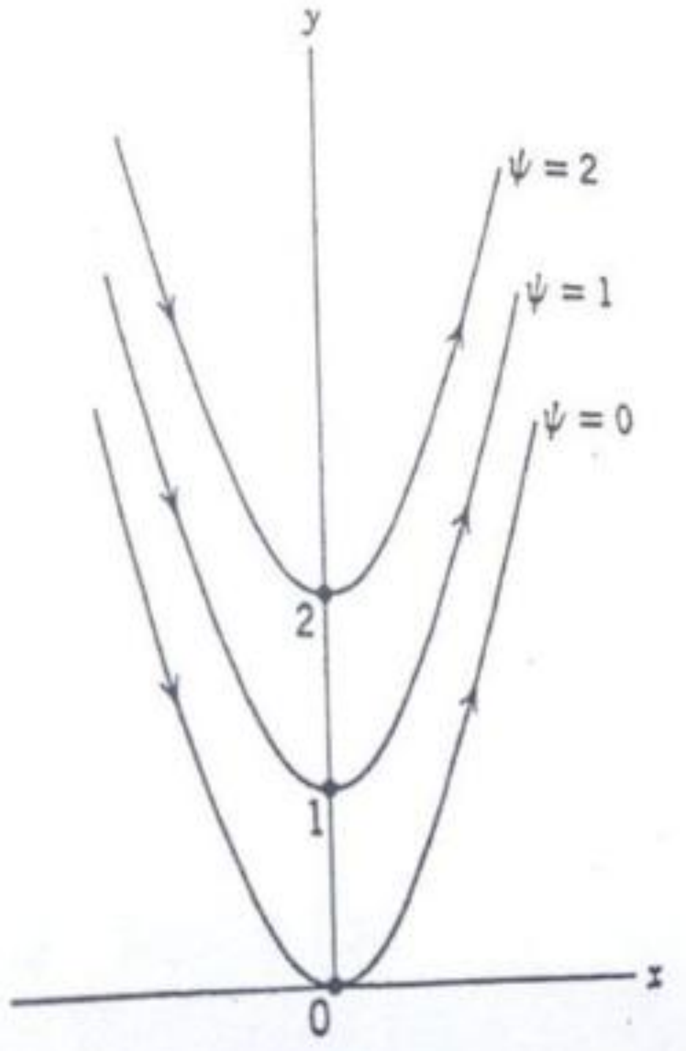
$$V = \sqrt{u^2 + v^2} = \sqrt{1 + 4x^2}$$

3 – *The equation of vorticity is*

$$\varepsilon = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) = \frac{1}{2}\left(\frac{\partial}{\partial x}(2x) - \frac{\partial}{\partial y}(1)\right)$$

$$= 1\ s^{-1}$$

Since $\varepsilon \neq 0$, the flowfield is rotational



Thank You