SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
An Autonomous Institution

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DEPARTMENT OF AEROSPACE ENGINEERING

23AST206 – AERODYNAMICS

II YEAR IV SEM

UNIT 2 – INCOMPRESSIBLE AND INVISCID FLOWS

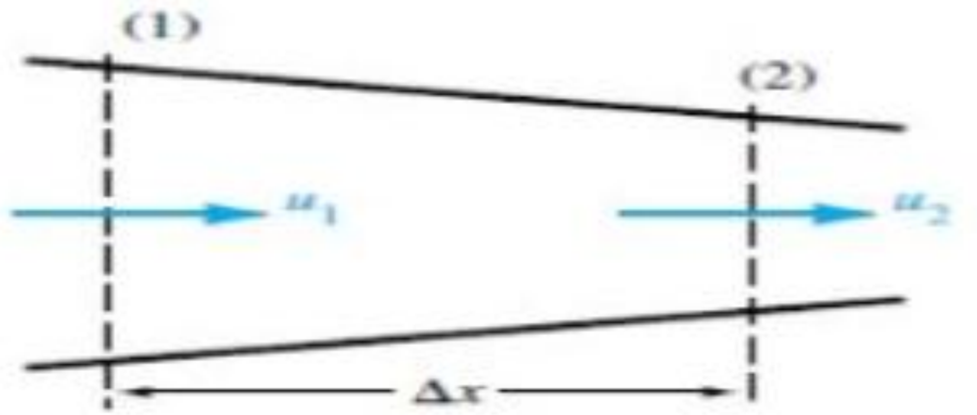
TOPIC – ROTATIONAL AND IRROTATIONAL FLOW

ROTATIONAL AND IRROTATIONAL FLOW

Rotational and Irrotational Flow

If curl $\underline{V}=0$ then the given flow field is irrotational. If curl $\underline{V}$ is non zero then the field is called rotational and curl $\underline{V}$ called the vorticity.

Convective Acceleration



EXAMPLE 1.5

Fluid flows through a contracting section of a duct, as in Fig. E1.5. A velocity probe inserted at section (1) measures a steady value $u_1 = 1$ m/s, while a similar probe at section (2) records a steady $u_2 = 3$ m/s. Estimate the fluid acceleration, if any, if $\Delta x = 10$ cm.

Solution

The flow is *steady* (not time-varying), but fluid particles clearly increase in velocity as they pass from (1) to (2). This is the concept of *convective acceleration* (Sec. 4.1). We may estimate the acceleration as a velocity change Δu divided by a time change $\Delta t = \Delta x / u_{avg}$:

$$a_x \approx \frac{\text{velocity change}}{\text{time change}} \approx \frac{u_2 - u_1}{\Delta x / [\frac{1}{2}(u_1 + u_2)]} = \frac{(3.0 - 1.0 \text{ m/s})(1.0 + 3.0 \text{ m/s})}{2(0.1 \text{ m})} \approx 40 \text{ m/s}^2 \qquad \textit{Ans.}$$

A simple estimate thus indicates that this seemingly innocuous flow is accelerating at 4 times

ROTATIONAL AND IRROTATIONAL FLOW

To write Newton's second law for an infinitesimal fluid system, we need to calculate the acceleration vector field **a** of the flow. Thus we compute the total time derivative of the velocity vector:

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \mathbf{i}\frac{du}{dt} + \mathbf{j}\frac{dv}{dt} + \mathbf{k}\frac{dw}{dt}$$

Since each scalar component (u, v, w) is a function of the four variables (x, y, z, t), we use the chain rule to obtain each scalar time derivative. For example,

$$\frac{du(x, y, z, t)}{dt} = \frac{\partial u}{\partial t} + \frac{\partial u}{\partial x}\frac{dx}{dt} + \frac{\partial u}{\partial y}\frac{dy}{dt} + \frac{\partial u}{\partial z}\frac{dz}{dt}$$

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But, by definition, dx/dt is the local velocity component u, and $dy/dt = v$, and $dz/dt = w$. The total derivative of u may thus be written in the compact form

$$\frac{du}{dt} = \frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} + w\frac{\partial u}{\partial z} = \frac{\partial u}{\partial t} + (\mathbf{V}\cdot\nabla)u \qquad (4.1)$$

Exactly similar expressions, with u replaced by v or w, hold for dv/dt or dw/dt. Summing these into a vector, we obtain the total acceleration:

$$\mathbf{a} = \frac{d\mathbf{V}}{dt} = \frac{\partial \mathbf{V}}{\partial t} + \left(u\frac{\partial \mathbf{V}}{\partial x} + v\frac{\partial \mathbf{V}}{\partial y} + w\frac{\partial \mathbf{V}}{\partial z}\right) = \frac{\partial \mathbf{V}}{\partial t} + (\mathbf{V}\cdot\nabla)\mathbf{V} \qquad (4.2)$$

Thank You