UNIT 4 Fourier Series and Fourier Transform

Unit - III FOURIER SERIES

PERIODIC FUNCTION:

A function $f(x)$ is said to be periodic if for all x, $f(x+T) = f(x)$, where T is a positive constant. The least value of $T > 0$ is called the period of $f(x)$.

For example:-

$$f(x) = \sin x = \sin(x + 2\pi) = \ldots$$

$\therefore \sin x$ is a periodic function with period 2π.

RESULTS:

$\sin 0 = 0$	$\sin n\pi = 0$	$\sin 2n\pi = 0$
$\cos 0 = 1$	$\cos n\pi = (-1)^n$	$\cos 2n\pi = 1$
$\cos(1+n)\pi = -(-1)^n$	$\cos(1-n)\pi = -(-1)^n$	$\cos \frac{n\pi}{2} = 0;\ n = 1, 3 \ldots$

FOURIER - EULER FORMULA:

The fourier series for the function $f(x)$ defined in the interval $(c, c+2\pi)$ with period 2π is given by.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where,

$$a_0 = \frac{1}{\pi} \int_c^{c+2\pi} f(x)\, dx \quad — (1)$$

$$a_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\cos nx \, dx \quad - (2)$$

$$b_n = \frac{1}{\pi}\int_{c}^{c+2\pi} f(x)\sin nx \, dx \quad - (3).$$

The values of a_0, a_n, b_n are known as Fourier - Euler formulae.

DIRICHLET'S CONDITIONS FOR THE EXPANSION OF FOURIER SERIES.

Any function $f(x)$ can be expressed as a Fourier series in $(c, c+2\pi)$ if

i) $f(x)$ must be periodic in $(c, c+2\pi)$.

ii) $f(x)$ must be single valued and finite in $(c, c+2\pi)$.

iii) $f(x)$ has finite ~~no~~ number of maxima and minima in $(c, c+2\pi)$

iv) $f(x)$ has finite number of finite discontinuities in $(c, c+2\pi)$.