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COIMBATORE-641 035, TAMIL NADU

#### **UNIT 4 Fourier Series and Fourier Transform**

District the Fourier Series of  $f(x) = \frac{\pi - x}{2}$ ;  $0 \le x \le 2\pi$ . Hence deduce that  $1 = \frac{1}{3} + \frac{1}{5} = \frac{\pi}{4}$ . f(x) is defined in (0,27).

The fourier series for f(x) in the interval
(0,27) is given by solution:  $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx - 0$  $a_0 = \frac{1}{\pi} \int f(x) dx$  $= \frac{1}{2\pi} \int_{0}^{2\pi} (\pi - x) dx$  $=\frac{1}{2\pi}\left[\pi\alpha-\frac{\pi^2}{2}\right]^{2\pi}$  $=\frac{5\pi}{1}\left[5\pi^{2}-\frac{5}{(5\pi)^{3}}-9-9\right]$  $= \frac{1}{2\pi} \left[ \frac{2\pi^2}{2} - \frac{4\pi^2}{2} \right] = \frac{1}{2\pi} (2\pi^2 - 2\pi^2) = \frac{1}{2\pi} (0)$ a0 = 0





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$$a_{n} = \frac{1}{\pi} \int_{0}^{\pi} f(x) \cos nx \, dx$$

$$= \frac{1}{\pi} \int_{0}^{\pi} (\frac{\pi}{2}x) \cos nx \, dx$$

$$= \frac{1}{2\pi} (\pi - 2) \left( \frac{\sin nx}{n} \right)_{0}^{2\pi} (o - 1) \left( -\frac{\cos nx}{n^{2}} \right)_{0}^{2\pi}$$

$$= \frac{-1}{2\pi n^{2}} (\cos nx)_{0}^{2\pi}$$

$$= \frac{-1}{2\pi n^{2}} (\cos nx)_{0}^{2\pi} (o)$$

$$= \frac{-1}{2\pi n^{2}} (\cos nx)_{0}^{2\pi} (o)$$

$$= \frac{-1}{2\pi n^{2}} (\sin nx)_{0}^{2\pi} (o)$$

$$= \frac{1}{2\pi n^{2}} \int_{0}^{\pi} f(x) \sin nx \, dx$$

$$= \frac{1}{\pi} \int_{0}^{\pi} (\pi - x) \sin nx \, dx$$

$$= \frac{1}{2\pi} \int_{0}^{\pi} (\pi - x) \sin nx \, dx$$



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$$= \frac{1}{2\pi} \left[ (J - x) \left( \frac{\cos nx}{n} \right) - (0 - 1) \left( \frac{\sin nx}{n^2} \right)^{2\pi} \right]$$

$$= \frac{-1}{2\pi\pi} \left[ (J - x) \cos nx \right]_0^{2\pi}$$

$$= \frac{-1}{2\pi\pi} \left[ (J - 2\pi) \cos 2n\pi - (J - 0) \cos 0 \right]$$

$$= \frac{-1}{2\pi\pi} \left[ -J(1) - J(1) \right] = \frac{-1}{2\pi\pi} \left[ -2\pi \right]$$

$$= \frac{1}{2\pi\pi} \left[ -J(1) - J(1) \right] = \frac{1}{2\pi\pi} \left[ -2\pi \right]$$

Substitute the values of  $a_0$ ,  $a_n$ ,  $b_n$  is  $a_n$ , we get  $f(a) = \frac{0}{2} + 0 + \sum_{n=1}^{\infty} \frac{1}{n} \sin n\alpha$ 

$$f(x) = \sum_{n=1}^{\infty} \frac{1}{n} s_n nx - 0$$

To deduce  $1-\frac{1}{3}+\frac{1}{5}-\cdots=\frac{\pi}{4}$ 

Since you contains sine tour, put  $x = \frac{\pi}{2}$  in Q\_ we get

$$f\left(\frac{\pi}{2}\right) = \sum_{N=1}^{\infty} \frac{1}{N} \sin \frac{n\pi}{2} - 3$$

To find  $f(\frac{\pi}{2})$ :  $f(x) = \frac{\pi-x}{2}, \quad 0 \quad \angle x \angle 2\pi$ 



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Here  $x = \frac{\pi}{2}$  (les inside (0,2). Hence f(x) continues at  $x = \frac{\pi}{2}$ 

$$\varphi\left(\frac{\pi}{2}\right) = \frac{\pi - \pi/2}{2} = \frac{\pi}{2} = \frac{\pi}{4}$$

Substitute the value of f(1) in 3 we get

$$\frac{37}{4} = \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{2n\pi}{2}$$

$$\therefore \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} = \frac{\pi}{4}$$

$$\sin \frac{\pi}{2} + \frac{1}{2} \sin \frac{2\pi}{2} + \frac{1}{3} \sin \frac{3\pi}{2} + \dots = \frac{\pi}{4}$$

Ans:

$$1 - \frac{1}{3} + \frac{1}{5} - \dots = \underbrace{\pi}_{A}$$

2) Find the Fourier review for  $f(x) = x (2\pi - x)$ .  $0 < x < 2\pi$ 

$$1)\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

$$ii) \frac{1}{1^2} * - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

$$111)$$
  $\frac{1}{4^2} + \frac{1}{3^2} + \frac{1}{5^2} \dots = \frac{\pi^2}{8}$ 



Walling Lines

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Soln: 
$$f(x)$$
 is defined in  $(0, 2\pi)$ 

The Fourier series for  $f(x)$  in the interval  $(0, 2\pi)$  is given by,

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos_2 nx + \sum_{n=1}^{\infty} b_n \sin_n x - 0$$

$$a_0 = \frac{1}{\pi} \int_0^{2\pi} (2\pi x - x^2) dx$$

$$= \frac{1}{\pi} \left[ 2\pi \frac{x^2}{2} - \frac{x^2}{3} \right]_0^{2\pi}$$

$$= \frac{1}{\pi} \left[ \pi (2\pi)^2 - \frac{(2\pi)^3}{3} - 0 - 0 \right]$$

$$= \frac{1}{\pi} \left[ 12\pi^3 - 8\pi^3 \right]$$

$$= \frac{1}{\pi} \int_0^{2\pi} f(x) \cos_n x dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} f(x) \cos_n x dx$$

$$= \frac{1}{\pi} \int_0^{2\pi} (2\pi x - x^2) \cos_n x dx$$



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$$= \frac{1}{\pi} \left[ (2\pi)_{2} - 2\pi \right] \frac{(2\pi)_{2}}{n} + (6-2) \left( \frac{1}{n^{2}} - 2\pi \right) \left( \frac{(2\pi)_{2}}{n^{2}} + (6-2) \left( \frac{(2\pi)_{2}}{n^{2}} - 2\pi \right) \left( \frac{(2\pi)_{2}}{n^{2}} - 2\pi \right) \right]$$

$$= \frac{1}{\pi} \left[ (2\pi - 4\pi) \left( \frac{(2\pi)_{2}}{n^{2}} - 2\pi \right) - 2\pi \right] \frac{(2\pi)_{2}}{n^{2}}$$

$$= \frac{1}{\pi} \left( -\frac{4\pi}{n^{2}} \right)$$

$$= \frac{1}{\pi} \left( -\frac{4\pi}{n^{2}} \right)$$

$$= \frac{1}{\pi} \left( -\frac{4\pi}{n^{2}} \right)$$

$$= \frac{1}{\pi} \left( (2\pi)_{2} - 2\pi \right) \frac{1}{n^{2}}$$

$$= \frac{1}{\pi} \left( (2\pi)_{2} - 2\pi \right) \frac{1}{n^{2}}$$

$$= \frac{1}{\pi} \left( (2\pi)_{2} - 2\pi \right) \frac{1}{n^{2}}$$

$$= \frac{1}{\pi} \left( (2\pi)_{2} - 2\pi \right) \frac{1}{n^{2}} + (-2\pi)_{2} \frac{1}{n^{2}} + (-2\pi)_{$$

 $=\frac{1}{2}\left[(2n\pi - x^2)\left(-\frac{\cos nx}{n}\right) - \frac{2(\cos nx)}{n}\right]$ 



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$$1 = \frac{1}{JI} \left[ \frac{-2}{n^3} + \frac{2}{n^3} \right]$$

Substitute the values of ao, an, b, in (1), we get

$$f(\alpha) = \frac{2\pi^2}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^2} \cos n \propto \frac{1}{n^2}$$

i) To deduce 
$$\frac{1}{1^2} + \frac{1}{2^2} + \dots = \frac{\pi^2}{6}$$

$$f(0) = \frac{2\pi^2}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^2} \cos 0$$

$$= \frac{2\pi^2}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^2} - 3$$

To find flo):

we have 
$$f(x) = 2\pi x - x^2$$
,  $0 < x < 2\pi$   
 $x = 0$  is end of  $(0, 2\pi)$ 

$$f(0) = f(0\pi) = 0$$



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Substitute the value of 
$$f(0)$$
 is (3) we get,
$$0 = \frac{2\pi^2}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^2}$$

$$4 \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{2\pi^2}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

ii) To deduce 
$$\frac{1}{1^2} = \frac{1}{2^2} + \frac{1}{3^2} = \frac{\pi^2}{19}$$

$$f(J) = \frac{2J^2}{3} - 4 \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nJ$$

$$f(\pi) = \frac{2\pi^2}{3} - 4 \lesssim \frac{1}{n^2} (-0)^n - 4$$

find f (IT):

$$\mathfrak{I} = \mathfrak{I}$$
 lies Inside  $(0, 2\mathfrak{I})$   
 $f(\mathfrak{I}) = 2\mathfrak{I}^2 - \mathfrak{I}^2$   
 $f(\mathfrak{I}) = \mathfrak{I}^2$ 

Substitute the value of f(J) in (4), we get



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$$\pi^{2} = \frac{2\pi^{2}}{3} - 4 \sum_{N=1}^{\infty} \frac{(-1)^{2}}{N^{2}}$$

$$-4 \sum_{N=1}^{\infty} \frac{(-1)^{N}}{N^{2}} = \pi^{2} - \frac{2\pi^{2}}{3}$$

$$= \frac{8\pi^{2} - 2\pi^{2}}{3}$$

$$= \frac{\pi^{2}}{3}$$

$$= \frac{$$



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# Homework sums:

D'find the Fourier series for f(x) = x is  $0 \angle x \angle 2\pi$ . Hence deduce  $1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$ 

Ans: 
$$f(x) = \pi - 2 \stackrel{\infty}{\leq} \frac{1}{h} \sin nx$$

(2) Find the Fourier series of  $f(x) = (JI - x)^2$ in (0, 211) of ferriodicity 2JI

Ans: 
$$f(x) = \frac{31^2}{3^2} + 4 \left[ \frac{\cos x}{1^2} + \frac{\cos 2x}{2^2} + \frac{\cos 3x}{3^2} + \cdots \right]$$

3) Find the Fourier series for f(x)=x2,

a) 
$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

$$b) \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - 1 \cdot \dots = \frac{\pi^2}{12}$$

c) 
$$\frac{1}{12} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

Ans:  $f(x) = \frac{4JI^2}{3} + 4 \sum_{n=1}^{\infty} \frac{1}{n^2} \cos nx$