| | |
|---|---|
| **CourseCode:** | **23MAT103** |
| **CourseName:** | **DIFFERENTIAL EQUATIONS AND TRANSFORMS** |
| **Year/Sem:** | **I/II** |

## QUESTION BANK
## UNIT IV FOURIER SERIES AND FOURIER TRANSFORMS
## FOURIER SERIES

**PART –A**

| Q.No | Question | Bloom's Taxonomy Level | Domain |
|---|---|---|---|
| 1. | State the Dirichlet's conditions for a function f(x) to be expanded as a Fourier series.<br>**Solution:**<br>(i) f(x) is periodic, single valued and finite.<br>(ii) f(x) has a finite number of discontinuities in any one period<br>(iii) f(x) has a finite number of maxima and minima.<br>(iv) f(x) and f'(x) are piecewise continuous. | BTL -1 | Remembering |
| 2. | Find the value of $a_0$ in the Fourier series expansion of $f(x)=e^x$ in $(0,2\pi)$.<br>**Solution:** $a_0 = \frac{1}{\pi}\int_0^{2\pi} f(x)\,dx = \frac{1}{\pi}\int_0^{2\pi} e^x\,dx = 0$. | BTL -1 | Remembering |
| 3. | If $(\pi - x)^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}\frac{cosnx}{n^2} in\ 0 < x < 2\pi$, then deduce that value of $\sum_{n=1}^{\infty}\frac{1}{n^2}$.<br>**Solution**: Put x=0, $\sum_{n=1}^{\infty}\frac{1}{n^2} = 6$. | BTL -1 | Remembering |
| 4. | Does $f(x)$ = tanx posses a Fourier expansion?<br>**Solution** No since tanx has infinite number of infinite discontinuous and not satisfying Dirichlet's condition. | BTL -2 | Understanding |
| 5. | Determine the value of $a_n$ in the Fourier series expansion of $f(x) = x^3$ in $(-\pi, \pi)$.<br>**Solution:** $a_n = 0$ since f(x) is an odd function | BTL -4 | Evaluating |
| 6. | Find the constant term in the expansion of $\cos^2 x$ as a Fourier series in the interval $(-\pi, \pi)$.<br>**Solution:** $a_0 = 1$ | BTL -2 | Understanding |

| | | | |
|---|---|---|---|
| **7**. | If f(x) is an odd function defined in (-l, l). What are the values of $a_0$ and $a_n$?<br>**Solution:** $a_n = 0 = a_0$ | **BTL -2** | Understanding |
| **8**. | If the function f(x) = x in the interval $0<x<2$ then find the constant term of the Fourier series expansion of the function f.<br>**Solution:** $a_0 = 4\pi$ | **BTL -2** | Understanding |

| | | | |
|---|---|---|---|
| **9.** | Expand f(x) =1 as a half range sine series in the interval (0, $\pi$ ).<br><br>**Solution:** The sine series of f(x) in (0, $\pi$ ) is given by<br>$f(x) = \sum_{n=1}^{\infty} b_n \sin nx$<br>where $b_n = \frac{2}{\pi}\int_0^{\pi} \sin nx\,dx = -\frac{2}{n\pi}\left[\cos nx\right]_0^{\pi} = 0$ if n is even<br>$= \frac{4}{n\pi}$ if n is odd<br>$f(x) = \sum_{n=odd}^{\infty} \frac{4}{n\pi}\sin nx = \frac{4}{\pi}\sum_{n=1}^{\infty} \frac{\sin(2n-1)x}{(2n-1)}.$ | **BTL -4** | Analyzing |
| **10.** | Find the value of the Fourier Series for<br>f(x) = 0 -c<x<0<br>= 1 0<x<c at x = 0<br>Solution: f(x) at x=0 is a discontinuous point in the middle.<br>$f(x)$ at $x = 0 = \frac{f(0-) + f(0+)}{2}$<br>$f(0-) = \lim_{h\to 0} f(0-h) = \lim_{h\to 0} 0 = 0$<br>$f(0+) = \lim_{h\to 0} f(0+h) = \lim_{h\to 0} 1 = 1$<br>$\therefore f(x)$ at x = 0 ➔ (0 + 1) / 2 = 1 / 2 = 0.5 | **BTL -3** | Applying |
| **11.** | What is meant by Harmonic Analysis?<br>Solution: The process of finding Euler constant for a tabular function is known as Harmonic Analysis. | **BTL -4** | Analyzing |
| **12.** | Find the constant term in the Fourier series corresponding to f (x) = $\cos^2 x$ expressed in the interval ($-\pi,\pi$).<br>Solution: Given $f(x) = \cos^2 x = \frac{1+\cos 2x}{2}$<br>W.K.T $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$<br>To find $a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} \cos^2 x\,dx = \frac{2}{\pi}\int_0^{\pi} \frac{1+\cos 2x}{2}\,dx = \frac{1}{\pi}\left[x + \frac{\sin 2x}{2}\right]_0^{\pi}$<br>$= \frac{1}{\pi}[(\pi + 0) - (0+0)] = 1.$ | **BTL -1** | Remembering |

| | | | |
|---|---|---|---|
| **13.** | Define Root Mean Square (or) R.M.S value of a function f(x) over the interval (a,b).<br>Solution: The root mean square value of f(x) over the interval (a,b) is defined as<br>R.M.S. $=\sqrt{\dfrac{\int_a^b [f(x)]^2\,dx}{b-a}}$ . | **BTL -3** | Applying |
| **14.** | Find the root mean square value of the function f(x) = x in the interval (0,$l$).<br>Solution: The sine series of f(x) in (a,b) is given by<br>R.M.S. $=\sqrt{\dfrac{\int_a^b [f(x)]^2\,dx}{b-a}} = \sqrt{\dfrac{\int_0^l [x]^2\,dx}{l-0}} = \dfrac{l}{\sqrt{3}}$ . | **BTL -1** | Remembering |
| **15.** | If f(x) = 2x in the interval (0,4), then find the value of $a_2$ in the Fourier series expansion.<br>Solution: $a_2 = \dfrac{2}{4}\int_0^4 2x\cos[\pi]x\,dx = 0.$ | **BTL -5** | Evaluating |
| **16.** | To which value, the half range sine series corresponding to f(x) = $x^2$ expressed in the interval (0,5) converges at x = 5?.<br>Solution: x = 2 is a point of discontinuity in the extremum.<br>$\therefore [f(x)]_{x=5} = \dfrac{f(0)+f(5)}{2} = \dfrac{[0]+[25]}{2} = \dfrac{25}{2}$ . | **BTL -4** | Analyzing |
| **17.** | If the Fourier Series corresponding to f(x) =x in the interval (0, 2 π) is $\dfrac{a_0}{2}+\sum_{n=1}^{\infty}(a_n\cos nx + b_n\sin nx)$ without finding the values of $a_0$, $a_n$ , $b_n$ find the value of $\dfrac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2)$ .<br>Solution: By Parseval's Theorem<br>$\dfrac{a_0^2}{2}+\sum_{n=1}^{\infty}(a_n^2+b_n^2) = \dfrac{1}{\pi}\int_0^{2\Pi}[f(x)]^2dx = \dfrac{1}{\pi}\int_0^{2\Pi}x^2dx = \dfrac{1}{\pi}\left[\dfrac{x^3}{3}\right]_0^{2\Pi}$<br>$= \dfrac{8}{3}\pi^2$ | **BTL -4** | Analyzing |

| | | | |
|---|---|---|---|
| **18.** | Obtain the first term of the Fourier series for the function f(x) $=x^2$ , $-\pi < x < \pi$.<br><u>Solution:</u> Given f(x) $=x^2$ ,is an even functionin $-\pi < x < \pi$.<br>Therefore,<br>$a_o = \frac{2}{\pi}\int_0^{\pi} f(x)dx = \frac{2}{\pi}\int_0^{\pi} x^2 dx = \frac{2}{\pi}\left[\frac{x^3}{3}\right]_0^{\pi} = \frac{2}{3}\pi^2.$ | **BTL -1** | Remembering |
| **19.** | Find the co-efficient $b_n$ of the Fourier series for the function f(x) = xsinx in (-2, 2).<br><u>Solution:</u> xsinx is an even function in (-2,2) . Therefore $b_n = 0$. | **BTL -4** | Analyzing |
| **20.** | Find the sum of the Fourier Series for<br>f(x) = x  0<x<1<br>= 2  1<x<2  at x = 1.<br><u>Solution:</u> f(x) at x=1 is a discontinuous point in the middle.<br>f(x) at x = 1 = $\frac{f(1-)+f(1+)}{2}$<br>$f(1-) = \lim_{h\to 0} f(1-h) = \lim_{h\to 0} 1-h = 1$<br>$f(1+) = \lim_{h\to 0} f(1+h) = \lim_{h\to 0} 2 = 2$<br>$\therefore$ f(x) at x = 1 ➔ (1 + 2) / 2 = 3 / 2 = 1.5 | **BTL -3** | Applying |

**PART – B**

| | | | |
|---|---|---|---|
| **1.(a)** | Obtain the Fourier's series of the function<br>$f(x) = \begin{cases} x & for \quad 0 < x < \pi \\ 2\pi - x & for \quad \pi < x < 2\pi \end{cases}.$<br>Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + ..... = \frac{\pi^2}{8}$ | **BTL -1** | Remembering |
| **1.(b)** | Find the Fourier's series of $f(x) = \lvert x \rvert$ in $-\pi < x < \pi$<br>And deduce that $\sum_{n=1}^{\infty} \frac{1}{(2n-1)^2} = \frac{\pi^2}{8}$ | **BTL -1** | Remembering |

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| f(x) | 9 | 18 | 24 | 28 | 26 | 20 |

| | | | |
|---|---|---|---|
| **2.(a)** | Find the Fourier's series expansion of period $2l$ for $f(x)=(l-x)^2$ in the range $(0,2l)$. Hence deduce that $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+....=\frac{\pi^2}{6}$ | **BTL -2** | Understanding |
| **2.(b)** | Find the Fourier series of periodicity $2\pi$ for $f(x)=x^2$ in $-\pi \le x \le \pi$. Hence deduce that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+....... = \frac{\pi^4}{90}$. | **BTL -2** | Understanding |
| **3.(a)** | Find the Fourier series upto second harmonic for the following data: | **BTL -1** | Remembering |
| **3.(b)** | Find the Fourier series of $f(x) = 2x - x^2$ in the interval $0 < x < 2$ | **BTL -1** | Remembering |
| **4.(a)** | Obtain the half range cosine series of the function $f(x)=\begin{cases} x & in \left(0,\frac{l}{2}\right) \\ l-x & \left(\frac{l}{2},l\right) \end{cases}$. | **BTL -4** | Analyzing |
| **4.(b)** | Find the half range sine series of the function $f(x)=x(\pi-x)$ in the interval (0 , Л) . | **BTL -3** | Applying |
| **5.(a)** | Determine the Fourier series for the function $f(x)=\lvert\sin x\rvert$ $in -\pi$ x . | **BTL -4** | Analyzing |
| **5.(b)** | Find the complex form of the Fourier series of $f(x) = e^{-ax}$ in (-l,l) | **BTL -1** | Remembering |

| | | | |
|---|---|---|---|
| **6.(a)** | Find the Fourier series for $f(x) = x\sin x$ *in* $(-\pi, \pi)$. | **BTL -2** | Remembering |
| **6.(b)** | Find the Fourier series expansion of $f(x) = x + x^2 \quad -2 \le x \le 2$. | **BTL - 2** | Remembering |
| **7.(a)** | Find the Fourier series for $f(x) = \begin{cases} x & (0, \pi/2) \\ \pi - x & (\pi/2, 2\pi). \end{cases}$ | **BTL - 4** | Analyzing |
| **7.(b)** | Find the Fourier series of $f(x) = x + x^2$ in (-l, l) with period 2l. | **BTL - 3** | Applying |
| **8.(a)** | Find the Fourier series as far as the second harmonic to represent the function f(x) with period 6, given in the following table. | **BTL -4** | Analyzing |

| **X** | **0** | **1** | **2** | **3** | **4** | **5** |
|---|---|---|---|---|---|---|
| **f(x)** | **9** | **18** | **24** | **28** | **26** | **20** |

| | | | |
|---|---|---|---|
| **8.(b)** | Find the complex form of the Fourier series of $f(x)=e^{-x}$ in $-1<x<1$ | **BTL -2** | Remembering |
| **9.(a)** | Find the half range cosine series for the function $f(x) = x(\pi - x)$ in $0 < x < \pi$. Deduce $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\dots=\frac{\pi^4}{90}$ | **BTL -2** | Remembering |
| **9.(b)** | Obtain the Fourier series to represent the function $f(x) = \lvert x \rvert, -\pi < x < \pi$ and deduce $\sum_{n=1}^{\infty}\frac{1}{(2n-1)^2}=\frac{\pi^2}{8}$ *(M/J 2012)* | **BTL -3** | Applying |
| **10.(a)** | Find the half range sine series of f(x) = lx-x in (0,l) ) | **BTL -1** | Remembering |
| **10.(b)** | Obtain the Fourier cosine series expansion of f(x) = x in 0<x<4. Hence deduce the value of $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\dots$ | **BTL -1** | Remembering |
| **11.(a)** | By using Cosine series show that $\frac{1}{1^4}+\frac{1}{2^4}+\frac{1}{3^4}+\dots=\frac{\pi^4}{96}$ for f(x) = x in $0 < x < \pi$ | **BTL -4** | Analyzing |
| **11.(b)** | Find the Fourier cosine series up to third harmonic to represent the function given by the following data: | **BTL -4** | Analyzing |

| X | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Y | 4 | 8 | 15 | 7 | 6 | 2 |

| | | | |
|---|---|---|---|
| **12.(a)** | Show that the complex form of Fourier series for the function $f(x)=e^{ax}$ $(-\pi,\pi)$ ) | **BTL -1** | Remembering |
| **12.(b)** | Find the complex form of the Fourier series of $f(x)=e^{-x}$ in $-1<x<1$. | **BTL -4** | Analyzing |
| **13.** | Calculate the first 3 harmonics of the Fourier of f(x) from the following data: | **BTL -4** | |

| x | 0 | 30 | 60 | 90 | 120 | 150 | 180 | 210 | 240 | 270 | | 330 | 360 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| f(x) | 1 8 | 1 1 | 0 3 | 0 16 | 0 5 | 1 3 | 2 16 | 1 25 | 1 3 | 1 52 | 1 76 | 2 | 1 8 |

An
aly
zin
g

| | | | |
|---|---|---|---|
| **14.(a)** | Find the complex form of the Fourier series of $f(x) = e^{-s}$ in $-1 < x < 1$. | **BTL -4** | Analyzing |
| **14.(b)** | Find the Fourier series up to the second harmonic from the following table. | **BTL - 4** | Analyzing |

| x | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| f(x) | 9 | 18 | 24 | 28 | 26 | 20 |

**FOURIER TRANSFORM**

**PART – A**

**CO Mapping : C214.2**

| Q.No | Questions | BT Level | Competence |
|---|---|---|---|
| 1 | Prove that $F[f(x-a)] = e^{ias}F(s)$ <br><br> **Proof:** <br> $F(f(x)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$ <br> $F(f(x-a)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x-a)e^{isx}dx,\quad put\ t = x-a;\ dt = dx$ <br> $x \to \pm\infty \Rightarrow t \to \pm\infty$ <br> $F(f(x-a)) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{is(t+a)}dt = e^{isa}\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(t)e^{ist}dt = e^{isa}F(s).$ | BTL-4 | Analyzing |
| 2 | Prove that $F(f(x)\cos ax) = \frac{1}{2}[F(s+a)+F(s-a)]$. <br><br> **Proof:** | BTL-1 | Remembering |

| | | | |
|---|---|---|---|
| | $F\left(f(x)\cos ax\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\cos ax\, e^{isx}\,dx$<br>$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)\frac{e^{iax}+e^{-iax}}{2}e^{isx}\,dx$<br>$= \frac{1}{2}\left(\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{i(s+a)x}\,dx + \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{i(s-a)x}\,dx\right)$<br>$= \frac{1}{2}\left[F(s+a)+F(s-a)\right].$ | | |
| **3** | Prove that $F_c\left(f(x)\sin ax\right) = \frac{1}{2}\left[F_s(s+a)+F_s(s-a)\right]$<br>**<u>Proof:</u>**<br>$F_c\left(f(x)\cos ax\right) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin ax\cos sx\,dx$<br>$= \frac{1}{2}\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\left(\sin(s+a)x+\sin(s-a)x\right)dx$<br>$= \frac{1}{2}\left(\sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} f(x)\sin(s+a)x\,dx + \sqrt{\frac{2}{\pi}}\int_{-\infty}^{\infty} f(x)\sin(s-a)x\,dx\right)$<br>$= \frac{1}{2}\left[F_s(s+a)+F_s(s-a)\right].$ | **BTL-2** | **Understanding** |
| **4** | Find the Fourier sine transform of $e^{-x}$, x >0.<br>**<u>Solution:</u>**<br>$F_s\left(f(x)\right) = \sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin sx\,dx = \sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-x}\sin sx\,dx$<br>$= \sqrt{\frac{2}{\pi}}\left[\frac{e^{-x}}{1+s^2}\left(-\sin sx - s\cos sx\right)\right]_0^{\infty} = \sqrt{\frac{2}{\pi}}\frac{s}{1+s^2}$ | **BTL-4** | **Analyzing** |

| | | | |
|---|---|---|---|
| 5 | Write the Fourier transform pair.<br>**Proof:**<br>$F(s)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} f(x)e^{isx}dx$<br>$f(x)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} F(s)e^{-isx}ds$ | **BTL-1** | **Remembering** |
| 6 | Find the Fourier sine transform of $\frac{1}{x}$.<br>**Solution:**<br>$F_s(f(x))=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(x)\sin sx\,dx=\sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{1}{x}\sin sx\,dx$<br>$put\ sx=\theta;\ sdx=d\theta;\quad =\sqrt{\frac{2}{\pi}}\int_0^{\infty}\frac{\sin\theta}{\theta}d\theta=\sqrt{\frac{2}{\pi}\frac{\pi}{2}}=\sqrt{\frac{\pi}{2}}.$ | **BTL-2** | **Understanding** |
| 7 | Find the Fourier cosine transform of $f(ax)$.<br>**Solution:**<br>$F_c(f(ax))=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(ax)\cos sx\,dx$<br>$put\ \ t=ax;\ dt=adx$<br>$=\sqrt{\frac{2}{\pi}}\int_0^{\infty} f(t)\cos\left(\frac{st}{a}\right)\frac{dt}{a}=\frac{1}{a}F_c\left(\frac{s}{a}\right).$ | **BTL-2** | **Understanding** |
| 8 | Find the Fourier Cosine transform of $e^{-ax}$.<br>**Solution:**<br>$F_c\left[e^{-ax}\right]=\sqrt{\frac{2}{\pi}}\int_0^{\infty} e^{-ax}\cos sx\,dx=\sqrt{\frac{2}{\pi}}\left[\frac{e^{-ax}}{a^2+s^2}(-a\cos sx+s\sin sx)\right]_0^{\infty}$<br>$=\sqrt{\frac{2}{\pi}}\frac{a}{a^2+s^2}.$ | **BTL-1** | **Remembering** |

| | | | |
|---|---|---|---|
| **9** | Find the Fourier transform of $f(x)=\begin{cases} e^{ikx}, & a<x<b \\ 0, & x<a, x>b \end{cases}$ <br> **<u>Solution:</u>** | **BTL-1** | **Remembering** |

$$F[f(x)] = \frac{1}{\sqrt{2\pi}}\int_a^b e^{ikx}e^{isx}dx = \frac{1}{\sqrt{2\pi}}\int_a^b e^{i(s+k)x}dx = \frac{1}{2\pi}\left[\frac{e^{i(s+k)x}}{i(s+k)}\right]_a^b$$

$$= \frac{1}{\sqrt{2\pi}}\left[\frac{e^{i(s+k)b} - e^{i(s+k)a}}{i(s+k)}\right].$$

| | | | |
|---|---|---|---|
| 10 | State convolution theorem.<br>***Solution* : If F(s) and G(s) are fourier transforms of f(x) and g(x) respectively then the fourier transform of the convolutions of f(x) and g(x) is the product of their fourier transform.** | BTL-1 | Remembering |
| 11 | Write the Fourier cosine transform pair?<br>***Solution :*** $F_c(s) = \frac{2}{\sqrt{\pi}}\int_0^\infty f(x)\cos sx\,dx$<br>$f(x) = \frac{\sqrt{2}}{\sqrt{\pi}}\int_0^\infty F_c(f(x)\cos sx\,ds$ | BTL-2 | Understanding |
| 12 | Write Fourier sine transform and its inversion formula?<br>***Solution :*** $F_s(s) = \frac{2}{\sqrt{\pi}}\int_0^\infty f(x)\sin sx\,dx$<br>$f(x) = \frac{\sqrt{2}}{\sqrt{\pi}}\int_0^\infty F_s(f(x)\sin sx\,ds$ | BTL-4 | Analyzing |
| 13 | State the modulation theorem in Fourier transform .<br>Solution : If F(s) is the Fourier transform of f(x) , then<br>F[f(x) cos ax] = 1/2 [F (s+a) +F(s-a). | BTL-4 | Analyzing |
| 14 | State the Parsevals identity on Fourier transform.<br>Solution : If F(s) is the Fourier transform of f(x), then<br>$\int_{-\infty}^{\infty}\lvert f(x)\rvert^2 dx = \int_{-\infty}^{\infty}\lvert F(s)\rvert^2 ds$ | BTL-4 | Analyzing |
| 15 | State Fourier Integral theorem .<br>***Solution : If f(x) is piecewise continuously differentiable & absolutely integrable in (-∞ , ∞ ) then***<br>f(x) = $\int^\infty\int^\infty f(t)e^{is(x-t)}dtds$ | ***This*** | $-\infty$ $-\infty$ ***is known as Fourier integral theorem*** |

| | | | |
|---|---|---|---|
| **BTL-1** | | **Remembering** | |
| **16** | Define self-reciprocal with respect to Fourier Transform.<br>Solution: If a transformation of a function f(x) is equal to f(s) then the function f(x) is called self-reciprocal | **BTL-4** | **Analyzing** |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |
| | | | |

| | | | |
|---|---|---|---|
| | **PART – B** | | |
| **1** | Find the Fourier transform of $f(x)=\begin{cases} a^2-x^2, & \lvert x\rvert\le a \\ 0, & \lvert x\rvert > a\end{cases}$. Hence evaluate $\int_0^\infty \frac{x\cos x-\sin x}{x^3}\cos\left(\frac{s}{2}\right)dx.$ | BTL-4 | Analyzing |
| **2** | Find the Fourier cosine transform of $f(x)=e^{-ax}, a>0$ and $g(x)=e^{-bx}, b>0$. Hence evaluate $\int_0^\infty \frac{dx}{(x^2+1)(x^2+9)}$. | **BTL-4** | **Analyzing** |
| **3** | Find the Fourier Transform of f(x) given by $f(x)=\begin{cases} a-\lvert x\rvert, & \lvert x\rvert\le a \\ 0, & \lvert x\rvert > a\end{cases}$. Hence show that $\int_0^\infty\left(\frac{\sin t}{t}\right)^2 dt=\frac{\pi}{2}$ $and$ $\int_0^\infty\left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}.$ | BTL-4 | Analyzing |

| | | | |
|---|---|---|---|
| **4** | Find the Fourier transform of $f(x)=\begin{cases}1, & for\ \lvert x\rvert \le a\\ 0, & for\ \lvert x\rvert > a > 0\end{cases}$ and using Parseval's identity prove that $\int_0^\infty \left(\frac{\sin t}{t}\right)^2 dt = \frac{\pi}{2}$. | **BTL-4** | **Analyzing** |
| **5** | Find the Fourier sine and cosine transform of $e^{-ax}$ and hence find the Fourier sine transform of $\frac{x}{x^2+a^2}$ and Fourier cosine transform of $\frac{1}{x^2+a^2}$. | BTL-4 | Analyzing |
| **6** | Find the Fourier cosine transform of $e^{-x^2}$. | **BTL-4** | **Analyzing** |
| **7** | Prove that $\frac{1}{\sqrt{x}}$ is self reciprocal under Fourier sine | BTL-4 | Analyzing |

| | | | | |
|---|---|---|---|---|
| | and cosine transforms. | | | |
| **8** | Evaluate $\int_0^\infty \frac{x^2\,dx}{(x^2+a^2)(x^2+b^2)}$ using Fourier | BTL-4 | Analyzing | |
| **9** | By finding the Fourier cosine transform of $f(x)=e^{-ax}(a>0)$ and using Parseval's identity for cosine transform evaluate $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$. | **BTL-3** | **Applying** | |

| | | | | |
|---|---|---|---|---|
| **10** | If $F_c(s)$ *and* $G_c(s)$ are the Fourier cosine transform of f(x) and g(x) respectively, then prove that $\int_0^\infty f(x)g(x)dx = \int_0^\infty F_c(s)G_c(s)ds.$ | **BTL-3** | **Applying** | |
| **11.** | Find the Fourier sine transform of $f(x)=\begin{cases} x, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0, & x>2. \end{cases}$ | BTL-4 | Analyzing | |
| **12.** | If $F_c(f(x))=F_c(s)$, prove that $F_c(F_c(x))=f(s)$. | **BTL-3** | **Applying** | |
| **13** | Use transform method to evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ | **BTL-3** | **Applying** | |