



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



## UNIT 4 Fourier Series and Fourier Transform

### Unit - III FOURIER SERIES

#### PERIODIC FUNCTION:

A function  $f(x)$  is said to be periodic if for all  $x$ ,  $f(x+T) = f(x)$ , where  $T$  is a positive constant. The least value of  $T > 0$  is called the period of  $f(x)$ .

For example:-

$$f(x) = \sin x = \sin(x + 2\pi) = \dots$$

$\therefore \sin x$  is a periodic function with period  $2\pi$ .

#### RESULTS:

$$\sin 0 = 0$$

$$\sin n\pi = 0$$

$$\sin 2n\pi = 0$$

$$\cos 0 = 1$$

$$\cos n\pi = (-1)^n$$

$$\cos 2n\pi = 1$$

$$\cos(1+n)\pi = -(-1)^n$$

$$\cos(1-n)\pi = -(-1)^n$$

$$\cos \frac{n\pi}{2} = 0; n=3, \dots$$

#### FOURIER - EULER FORMULA:

The Fourier series for the function  $f(x)$  defined in the interval  $(c, c+2\pi)$  with period  $2\pi$  is given by.

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos nx + \sum_{n=1}^{\infty} b_n \sin nx$$

where,

$$a_0 = \frac{1}{\pi} \int_c^{c+2\pi} f(x) dx \quad \text{--- (1)}$$



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$$a_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \cos nx \, dx \quad - (2)$$

$$b_n = \frac{1}{\pi} \int_c^{c+2\pi} f(x) \sin nx \, dx \quad - (3)$$

The values of  $a_0$ ,  $a_n$ ,  $b_n$  are known as Fourier-Euler formulae.

### DIRICHLET'S CONDITIONS FOR THE EXPANSION OF FOURIER SERIES.

Any function  $f(x)$  can be expressed as a Fourier series in  $(c, c+2\pi)$  if

- i)  $f(x)$  must be periodic in  $(c, c+2\pi)$ .
- ii)  $f(x)$  must be single valued and finite in  $(c, c+2\pi)$ .
- iii)  $f(x)$  has finite number of maxima and minima in  $(c, c+2\pi)$ .
- iv)  $f(x)$  has finite number of finite discontinuities in  $(c, c+2\pi)$ .