PROBLEMS UNDER FOURIER SINE AND FOURIER COSINE TRANSFORMS:

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx \, dx = F_s(s)$$

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_s(s) \sin sx \, ds.$$

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \, ds = F_c(s)$$

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^\infty F_c(s) \cos sx \, ds.$$

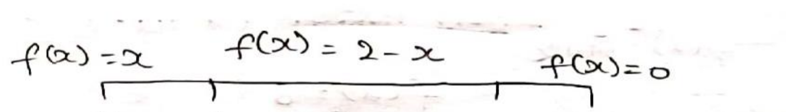
(1) Find the Fourier cosine transform for

$$f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2-x & ; 1 < x < 2 \\ 0 & ; x > 2 \end{cases}$$

Solution:

The Fourier cosine transform for $f(x)$ is

$$F_c[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \cos sx \, dx$$



$$= \sqrt{\frac{2}{\pi}} \left[\int_0^1 x \cos sx \, dx + \int_1^2 (2-x) \cos sx \, dx \right.$$

$$+ \int_2^{\infty} 0 \cos sx \, dx \Big]$$

$$= \sqrt{\frac{2}{\pi}} \left[x\left(\frac{\sin sx}{s}\right) - (1)\left(-\frac{\cos sx}{s^2}\right) \right]_0^1$$

$$+ \left[(2-x)\left(\frac{\sin sx}{s}\right) - (-1)\left(-\frac{\cos sx}{s^2}\right) \right]_1^2$$

$$= \sqrt{\frac{2}{\pi}} \left[\frac{\sin s}{s} + \frac{\cos s}{s^2} - 0 - \frac{1}{s^2} - \frac{\cos 2s}{s^2} - \frac{\sin s}{s} + \frac{\cos s}{s^2} \right]$$

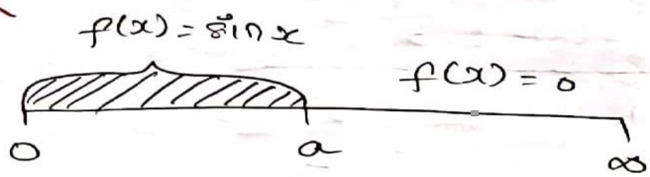
$$= \sqrt{\frac{2}{\pi}} \left[\frac{\cos s}{s^2} - \frac{1}{s^2} - \frac{\cos 2s}{s^2} + \frac{\cos s}{s^2} \right]$$

$$\boxed{F_c[f(x)] = \sqrt{\frac{2}{\pi}} \left(\frac{2\cos s - \cos 2s - 1}{s^2} \right)}$$

② Find the Fourier sine transform for

$$f(x) = \begin{cases} \sin x & 0 < x < a \\ 0 & x > a \end{cases}$$

Solution:



The Fourier sine transform for $f(x)$ is

$$F_s[f(x)] = \sqrt{\frac{2}{\pi}} \int_0^\infty f(x) \sin sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \left[\int_0^a \sin x \sin sx \, dx + \int_0^\infty 0 \sin sx \, dx \right]$$

$$= \sqrt{\frac{2}{\pi}} \int_0^a \sin x \sin sx \, dx$$

$$\boxed{\because \sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}}$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{2} \int_0^a [\cos(1-s)x - \cos(1+s)x] \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{\sin(1-s)x}{1-s} - \frac{\sin(1+s)x}{1+s} \right]_0^a$$

$$= \frac{1}{\sqrt{2\pi}} \left[\frac{\sin(1-s)a}{1-s} - \frac{\sin(1+s)a}{1+s} \right]$$