



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



## PROBLEMS UNDER FOURIER SINE AND COSINE TRANSFORMS:

$$F_s [f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx \, dx = F_s(s)$$

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_s(s) \sin sx \, ds$$

$$F_c [f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx \, ds = F_c(s)$$

$$f(x) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} F_c(s) \cos sx \, ds$$

① Find the Fourier cosine transform for

$$f(x) = \begin{cases} x & ; 0 < x < 1 \\ 2-x & ; 1 < x < 2 \\ 0 & ; x > 2 \end{cases}$$

Solution:

The Fourier cosine transform for  $f(x)$  is

$$F_c [f(x)] = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx \, dx$$

$f(x) = x$	$f(x) = 2-x$	$f(x) = 0$
0	1	2
-----		
		$\infty$

$$= \sqrt{\frac{2}{\pi}} \left[ \int_0^1 x \cos sx \, dx + \int_1^2 (2-x) \cos sx \, dx \right]$$



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

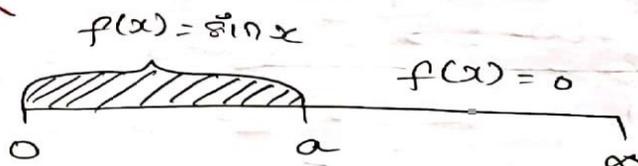
COIMBATORE-641 035, TAMIL NADU



$$\begin{aligned} & + \int_0^{\infty} \cos sx \, dx \\ & = \sqrt{\frac{2}{\pi}} \left[ x \left( \frac{\sin sx}{s} \right) - (1) \left( -\frac{\cos sx}{s^2} \right) \right]_0^{\infty} \\ & \quad + \left[ (2-x) \left( \frac{\sin sx}{s} \right) - (1) \left( -\frac{\cos sx}{s^2} \right) \right]_1^2 \\ & = \sqrt{\frac{2}{\pi}} \left[ \frac{\sin s}{s} + \frac{\cos s}{s^2} - 0 - \frac{1}{s^2} - \frac{\cos 2s}{s^2} - \frac{\sin s}{s} + \frac{\cos s}{s^2} \right] \\ & = \sqrt{\frac{2}{\pi}} \left[ \frac{\cos s}{s^2} - \frac{1}{s^2} - \frac{\cos 2s}{s^2} + \frac{\cos s}{s^2} \right] \\ & \boxed{F_c [f(x)] = \sqrt{\frac{2}{\pi}} \left( \frac{2 \cos s - \cos 2s - 1}{s^2} \right)} \end{aligned}$$

② Find the Fourier sine transform for  
 $f(x) = \begin{cases} \sin x & 0 < x < a \\ 0 & x > a \end{cases}$

Solution:



The Fourier sine transform for  $f(x)$  is



# SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



$$\begin{aligned}F_s[f(x)] &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx \, dx \\&= \sqrt{\frac{2}{\pi}} \left[ \int_0^a \sin x \sin sx \, dx + \int_0^{\infty} 0 \sin sx \, dx \right] \\&= \sqrt{\frac{2}{\pi}} \int_0^a \sin x \sin sx \, dx\end{aligned}$$

$$\therefore \sin A \sin B = \frac{\cos(A-B) - \cos(A+B)}{2}$$

$$= \sqrt{\frac{2}{\pi}} \cdot \frac{1}{2} \int_0^a [\cos(1-s)x - \cos(1+s)x] \, dx$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{\sin(1-s)x}{1-s} - \frac{\sin(1+s)x}{1+s} \right]_0^a$$

$$= \frac{1}{\sqrt{2\pi}} \left[ \frac{\sin(1-s)a}{1-s} - \frac{\sin(1+s)a}{1+s} \right]$$