



SNS COLLEGE OF TECHNOLOGY

Coimbatore-35
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DEPARTMENT OF ARTIFICIAL INTELLIGENCE AND MACHINE LEARNING

19ITB302-Cryptography and Network Security

UNIT-2 NUMBER THEORY AND PUBLIC KEY CRYPTOSYSTEMS



Random Numbers

1. Randomness

Sequence of numbers be random in some well- defined statistical sense.

Uniform distribution: The distribution of bits in the sequence should be uniform; that is, the frequency of occurrence of ones and zeros should be approximately equal.

2. Unpredictability

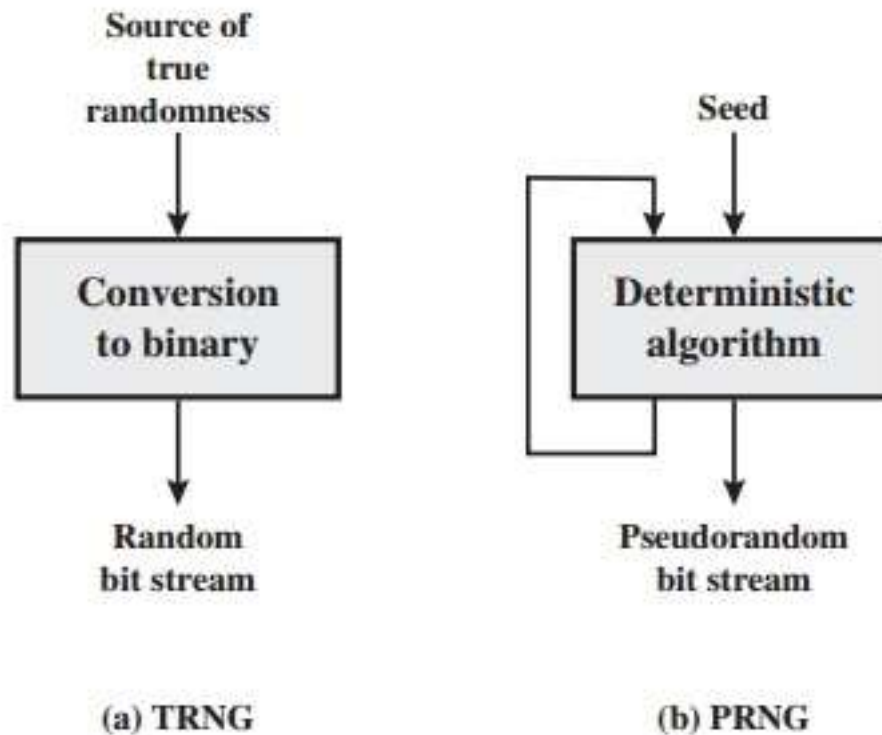
The values are uniformly distributed over a defined interval or set, and it is impossible to predict future values based on past or present ones.



Pseudorandom Number Generator



Pseudo-random numbers are generated using deterministic algorithms and appear random



Pseudorandom Number Generator

$$\begin{array}{rcl} & \text{PTS} & X_1 \ X_2 \ X_3 \ X_4 \ \dots \ X_n \\ \oplus & \text{KS} & K_1 \ K_2 \ K_3 \ K_4 \ \dots \ K_n \\ \hline & \text{CTS} & Y_1 \ Y_2 \ Y_3 \ Y_4 \ \dots \ Y_n \end{array}$$



Alice



Bob

$$\begin{array}{rcl} \text{CTS} & Y_1 \ Y_2 \ Y_3 \ Y_4 \ \dots \ Y_n \\ \text{KS} & K_1 \ K_2 \ K_3 \ K_4 \ \dots \ K_n \\ \hline \text{PTS} & X_1 \ X_2 \ X_3 \ X_4 \ \dots \ X_n \end{array}$$



Linear Congruential Generators

- $X_{i+1} = (a * X_i + c) \bmod m$
- $R_i = X_i / m$
- X_0 = Starting Seed value
- a is the multiplier
- C is the increment
- m is the modulus



Example

Given values

$$\blacktriangleright X_0=27, a=17, c=43, m=100$$

$$\blacktriangleright X_{i+1}=(aX_i+c)\bmod m$$

$$\blacktriangleright X_1=(17*27+43)\bmod 100$$
$$=502 \bmod 100$$

$$X_1=2$$

$$X_2=(17*2+43)\bmod 100$$
$$=77 \bmod 100$$

$$X_2=77$$

$$\bullet X_3=(17*77+43)\bmod 100$$
$$=1352 \bmod 100$$
$$=52$$

$$X_4=(17*52+43)\bmod 100$$
$$=927 \bmod 100$$
$$=27$$

$$X_5=(17*27+43)\bmod 100$$
$$=502 \bmod 100$$
$$=2$$



$$R_i = X_i / m$$

$$R_1 = 2/100 = 0.02$$

$$R_2 = 77/100 = 0.77$$

$$R_3 = 52/100 = 0.52$$

$$R_4 = 27/100 = 0.27$$

$$R_5 = 2/100 = 0.02$$



Blum Blum Shub Generator

- It was created by Lenore Blum, Manuel Blum and Michael Shub in 1968.
- Cryptographically secure pseudorandom generator
- Choose two prime numbers p, q such that both have a remainder of 3 when divided by 4
- Next compute $n = p * q$ (**eg: $p = 7, q = 11$**)
- Choose a random number s , such that s is relatively prime to n (**any integer that is not divisible by 7 or 11 will be relatively prime to 77.**)

Algorithm

$$X_0 = s^2 \bmod n$$

For $i = 1$ to infinity

$$X_i = (X_{i-1} - 1)^2 \bmod n$$

$$B_i = X_i \bmod 2$$



Division Algorithm

The notation $b \mid a$ is commonly used to mean b divides a . Also, if $b \mid a$, we say that b is a **divisor** of a .

Given any positive integer b and a ,

if we divide a by b , we get an integer quotient q and an integer remainder r that obey the following relationship:

$$a = qb + r \text{ where } 0 < r < b ; q = a/b$$

Example

$$a = 21, b = 2$$

$$a = 10 * 2 + 1 (r = 1 \text{ and } r \text{ is between } 0 \text{ and } 2)$$



Euclidean Algorithm



The Euclidean algorithm is a way to find the greatest common divisor of two positive integers.
GCD of two numbers is the largest number that divides both of them

The Algorithm

The Euclidean Algorithm for finding $\text{GCD}(A, B)$ is as follows:

- If $A = 0$ then $\text{GCD}(A, B) = B$, since the $\text{GCD}(0, B) = B$, and we can stop.
- If $B = 0$ then $\text{GCD}(A, B) = A$, since the $\text{GCD}(A, 0) = A$, and we can stop.
- Write A in quotient remainder form ($A = B \cdot Q + R$)
- Find $\text{GCD}(B, R)$ using the Euclidean Algorithm since $\text{GCD}(A, B) = \text{GCD}(B, R)$