



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Coimbatore-641035.



UNIT-IV COMPLEX INTEGRATION

SINGULARITIES&RESIDUE

Singular point :-

A point $z=a$ is said to be a singular point if $f(z)$ is not analytic at that point.

$f(z) = 1/z$, which is not analytic at $z=0$.

types of singularities

i) Isolated singularities :-

A point $z=a$ is said to be isolated singularities if these are few $\rightarrow f(z)$ is not analytic at

$z=0$.

$\rightarrow f(z)$ is analytic at all points for some neighbourhood of that point.

ex:- $f(z) = \frac{z}{(z-1)(z-2)}$

ii) Pole :-

A point $z=a$ is said to be a pole of order n , if we can find the positive integer such that $\lim_{z \rightarrow a} (z-a)^n f(z) \neq 0$



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Eg: $\frac{z-1}{(z-1)^2(z-3)^4}$

8) Essential singularities: A point $z=a$ is said to be an essential singularity if Laurent series of $f(z)$ about $z=a$ which possess infinite number of terms in the principle part (terms that contain (-ve) powers) being non-zero.

Eg: $e^{1/z}$ at $z=0$

4) Removable singularity: the singular point $z=a$ is called a removable singularity if $\lim_{z \rightarrow a} f(z)$ exist.

Eg: $\frac{\sin z}{z}$

At the point $z=0$, the function is defined as $e^x = 1 + x + \frac{x^2}{2!} + \dots$ hence both $\sin z = z - \frac{z^3}{3!} + \frac{z^5}{5!} + \dots$ and $\frac{\sin z}{z} = 1 + \frac{z^2}{2!} + \frac{(1/z)^2}{2!} + \dots$ classify the singularities of the following

- $\frac{8\pi n z - z}{z^3}$ - Removable
- $\frac{\tan z}{z}$ - Removable
- $8\pi n \left(\frac{1}{z+1}\right)$ - essential



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Residue :-

then $\text{[Res } f(z)]_{z=a} = \lim_{z \rightarrow a} (z-a) f(z)$ if $f(z) = a$ is pole of order 1,

then $\text{[Res } f(z)]_{z=a} = \frac{1}{(m-1)!} \lim_{z \rightarrow a} \frac{d^{m-1}}{dz^{m-1}} (z-a)^m f(z)$ if $f(z) = a$ is pole of order m ,

1, find the residue of $f(z) = \frac{z+1}{z(z-2)}$ at the point $z=2$.

Soln:-

$$\text{Here } z(z-2) = 0$$

$$z=0 \text{ and } z-2=0 \\ z=2$$

$z=0$ is a pole of order 1

$z=2$ is a pole of order 1

$$[\text{Res } f(z)] = \lim_{z \rightarrow 2} (z-2) \frac{z+1}{z(z-2)}$$

$$= \lim_{z \rightarrow 2} \frac{z+1}{z} = \frac{2+1}{2} = \frac{3}{2},$$