



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

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COIMBATORE-641 035, TAMIL NADU



UNIT 4 Fourier Series and Fourier Transform

PROBLEMS UNDER FOURIER SERIES IN (0, 2l)

① Find the Fourier series for $f(x) = x^2$ in $0 < x < 2l$. Hence deduce

$$i) \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

$$ii) \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

$$iii) \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

Solution:

$f(x)$ is defined in $(0, 2l)$

The Fourier series for $f(x)$ in the interval $(0, 2l)$ is given by

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \quad \text{--- (1)}$$

$$a_0 = \frac{1}{l} \int_0^{2l} f(x) dx$$

$$= \frac{1}{l} \int_0^{2l} x^2 dx$$

$$= \frac{1}{l} \left[\frac{x^3}{3} \right]_0^{2l} = \frac{1}{l} \left[\frac{8l^3}{3} \right]$$

$$\boxed{a_0 = \frac{8l^2}{3}}$$



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UNIT 4 Fourier Series and Fourier Transform

$$\begin{aligned}a_n &= \frac{1}{l} \int_0^{2l} f(x) \cos \frac{n\pi x}{l} dx \\&= \frac{1}{l} \int_0^{2l} x^2 \cos \frac{n\pi x}{l} dx \\&= \frac{1}{l} \left[x^2 \left(\frac{\sin \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right) - 2x \left(\frac{-\cos \frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}} \right) + 2 \left(\frac{-\sin \frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}} \right) \right]_0^{2l} \\&= \frac{1}{l} \frac{2l^2}{n^2\pi^2} \left[x \cos \frac{n\pi x}{l} \right]_0^{2l} \\&= \frac{2l}{n^2\pi^2} \left(2l \cos \frac{2n\pi l}{l} - 0 \cos 0 \right) \\&= \frac{2l}{n^2\pi^2} (2l - \cos 2n\pi) \\&\boxed{a_n = \frac{4l^2}{n^2\pi^2}}\end{aligned}$$

$$\begin{aligned}b_n &= \frac{1}{l} \int_0^{2l} f(x) \sin \frac{n\pi x}{l} dx \\&= \frac{1}{l} \int_0^{2l} x^2 \sin \frac{n\pi x}{l} dx \\&= \frac{1}{l} \left[x^2 \left(\frac{-\cos \frac{n\pi x}{l}}{\frac{n\pi}{l}} \right) - 2x \left(\frac{-\sin \frac{n\pi x}{l}}{\frac{n^2\pi^2}{l^2}} \right) + 2 \left(\frac{\cos \frac{n\pi x}{l}}{\frac{n^3\pi^3}{l^3}} \right) \right]_0^{2l}\end{aligned}$$



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$$\begin{aligned}
&= \frac{1}{L} \left[x^3 \left(\frac{-\cos \frac{n\pi x}{L}}{\frac{n\pi}{L}} \right) + 2 \left(\frac{\cos \frac{n\pi x}{L}}{\frac{n^3\pi^3}{L^2}} \right) \right]_0^{2L} \\
&= \frac{1}{L} \left[4L^2 \left(\frac{-\cos 2n\pi}{n\pi} \right) + \left(\frac{2 \cos 2n\pi}{n^3\pi^3} \right) - \left(0 + \frac{2 \cos 0}{n^3\pi^3} \right) \right] \\
&= \frac{1}{L} \left[\frac{-4L^2}{\frac{n\pi}{L}} + \frac{2}{\frac{n^3\pi^3}{L^3}} - \frac{2}{\frac{n^3\pi^3}{L^3}} \right]
\end{aligned}$$

$$b_n = \frac{-4L^2}{n\pi}$$

Substitute the values of a_0, a_n, b_n in ①, we get

$$f(x) = \frac{4L^2}{3} + \sum_{n=1}^{\infty} \frac{4L^2}{n^2\pi^2} \cos \frac{n\pi x}{L} + \sum_{n=1}^{\infty} \frac{-4L^2}{n\pi} \sin \frac{n\pi x}{L} \quad \text{--- ②}$$

i) To deduce $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} \dots = \frac{\pi^2}{6}$

Put $x=0$ in ②, we get

$$\text{②} \Rightarrow f(0) = \frac{4L^2}{3} + \sum_{n=1}^{\infty} \frac{4L^2}{n^2\pi^2} \cos 0 - \sum_{n=1}^{\infty} \frac{4L^2}{n\pi} \sin 0$$

$$f(0) = \frac{4L^2}{3} + \sum_{n=1}^{\infty} \frac{4L^2}{n^2\pi^2} \quad \text{--- ③}$$

To find $f(0)$:

When have $f(x) = x^2$ in $0 < x < 2L$.



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$x=0$ is the end of $(0, 2l)$

$$f(0) = 0$$

$$f(2l) = (2l)^2 = 4l^2$$

$$f(0) \neq f(2l)$$

Here $x=0$ is a point of discontinuity

$$\therefore f(0) = \frac{f(0) + f(2l)}{2}$$

$$= \frac{0 + 4l^2}{2}$$

$$f(0) = 2l^2$$

Substitute the value of $f(0)$ in (3), we get

$$2l^2 = \frac{4l^2}{3} + \sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2}$$

$$2l^2 - \frac{4l^2}{3} = \sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2}$$

$$\sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2} = \frac{6l^2 - 4l^2}{3}$$

$$\frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{2l^2}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6}$$

(a)



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ii) To deduce $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$

Put $x=1$ in (2), we get

$$f(1) = \frac{4 \cdot 1^2}{3} + \sum_{n=1}^{\infty} \frac{4 \cdot 1^2}{n^2 \pi^2} \cos \frac{n\pi \cdot 1}{1} - \sum_{n=1}^{\infty} \frac{4 \cdot 1^2}{n^2 \pi^2} \sin \frac{n\pi \cdot 1}{1}$$

$$f(1) = \frac{4 \cdot 1^2}{3} + \sum_{n=1}^{\infty} \frac{4 \cdot 1^2 (-1)^n}{n^2 \pi^2} - 0$$

$$f(1) = \frac{4 \cdot 1^2}{3} + \sum_{n=1}^{\infty} \frac{4 \cdot 1^2 (-1)^n}{n^2 \pi^2} \quad \text{--- (4)}$$

To find $f(1)$:

$x=1$ lies inside $(0, 2\pi)$. Hence $x=1$ is a point of continuity

$$\therefore f(1) = 1^2$$

Substitute the value of $f(x)$ in (4) we get

$$1^2 = \frac{4 \cdot 1^2}{3} + \sum_{n=1}^{\infty} \frac{4 \cdot 1^2 (-1)^n}{n^2 \pi^2}$$

$$\sum_{n=1}^{\infty} \frac{4 \cdot 1^2 (-1)^n}{n^2 \pi^2} = 1^2 - \frac{4 \cdot 1^2}{3}$$

$$\frac{4 \cdot 1^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} (-1)^n = \frac{-1^2}{3}$$

$$\sum_{n=1}^{\infty} \frac{1}{n^2} (-1)^n = \frac{-\pi^2}{12}$$

$$\frac{(-1)^1}{1^2} + \frac{(-1)^2}{2^2} + \frac{(-1)^3}{3^2} + \dots = \frac{-\pi^2}{12}$$



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$$-\frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{3^2} + \dots = -\frac{\pi^2}{12}$$

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12} \quad \text{--- (b)}$$

iii) To deduce: $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

Add (a) and (b) we get,

$$\frac{1}{1^2} + \frac{1}{1^2} + \frac{1}{2^2} - \frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{6} + \frac{\pi^2}{12}$$

$$2\left(\frac{1}{1^2}\right) + 2\left(\frac{1}{3^2}\right) + \dots = \frac{18\pi^2}{72}$$

Ans: $\boxed{\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}}$

② Find the Fourier series for
 $f(x) = \begin{cases} 1-x, & 0 < x < 1 \\ 0, & 1 < x < 2 \end{cases}$ hence deduce.

i) $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$

ii) $1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$

Solution:

$f(x)$ is defined in $(0, 2)$

The Fourier series for $f(x)$ in the interval $(0, 2)$ is given by.



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$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \quad \text{--- (1)}$$

$$a_0 = \frac{1}{l} \int_0^{2l} f(x) dx$$

$$= \frac{1}{l} \left[\int_0^l f(x) dx + \int_l^{2l} f(x) dx \right]$$

$$= \frac{1}{l} \left[\int_0^l (1-x) dx + \int_l^{2l} 0 dx \right]$$

$$= \frac{1}{l} \left[lx - \frac{x^2}{2} \right]_0^l$$

$$= \frac{1}{l} \left[l^2 - \frac{l^2}{2} \right]$$

$$= \frac{1}{l} \left(\frac{l^2}{2} \right)$$

$$\boxed{a_0 = \frac{l}{2}}$$

$$a_n = \frac{1}{l} \int_0^{2l} f(x) \cos \frac{n\pi x}{l} dx$$

$$= \frac{1}{l} \left[\int_0^l f(x) \cos \frac{n\pi x}{l} dx + \int_l^{2l} f(x) \cos \frac{n\pi x}{l} dx \right]$$

$$= \frac{1}{l} \left[\int_0^l (1-x) \cos \frac{n\pi x}{l} dx + \int_l^{2l} 0 dx \right]$$



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$$= \frac{1}{\lambda} \left[(1-x) \left(\frac{\sin \frac{n\pi x}{\lambda}}{\frac{n\pi}{\lambda}} \right) - (-1) \left(\frac{-\cos \frac{n\pi x}{\lambda}}{\left(\frac{n\pi}{\lambda}\right)^2} \right) \right]_0^{\lambda}$$

$$= \frac{-1}{\lambda} \left(\frac{\lambda}{n\pi} \right)^2 \left[\cos \frac{n\pi x}{\lambda} \right]_0^{\lambda}$$

$$= \frac{-\lambda}{n^2 \pi^2} \left[\cos \frac{n\pi \lambda}{\lambda^2} - \cos 0 \right]$$

$$= \frac{-\lambda}{n^2 \pi^2} \left[(-1)^n - 1 \right]$$

$$= \frac{\lambda}{n^2 \pi^2} \left[1 - (-1)^n \right]$$

$$a_n = \begin{cases} \frac{2\lambda}{n^2 \pi^2} & ; n = 1, 3, 5, \dots \\ 0 & ; n = 2, 4, 6, \dots \end{cases}$$

$$b_n = \frac{1}{\lambda} \int_0^{2\lambda} f(x) \sin \frac{n\pi x}{\lambda} dx$$

$$= \frac{1}{\lambda} \left[\int_0^{\lambda} (1-x) \sin \frac{n\pi x}{\lambda} dx + \int_{\lambda}^{2\lambda} 0 \right]$$

$$= \frac{1}{\lambda} \left[(1-x) \left(\frac{-\cos \frac{n\pi x}{\lambda}}{\frac{n\pi}{\lambda}} \right) - (-1) \left(\frac{-\sin \frac{n\pi x}{\lambda}}{\left(\frac{n\pi}{\lambda}\right)^2} \right) \right]_0^{\lambda}$$



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$$\begin{aligned}
&= \frac{-1}{2} \cdot \frac{1}{n\pi} \left[(1-x) \left(\cos \frac{n\pi x}{2} \right) \right]_0^2 \\
&= \frac{-1}{n\pi} \left[(0) \left(\cos \frac{n\pi}{2} \right) - (1) \left(\cos 0 \right) \right] \\
&= \frac{-1}{n\pi} [0 - 1]
\end{aligned}$$

$$b_n = \frac{1}{n\pi}$$

Substitute the values of a_0 , a_n , b_n in ①

$$f(x) = \frac{1}{4} + \sum_{n=1,3}^{\infty} \frac{2x}{n^2\pi^2} \cos \frac{n\pi x}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \sin \frac{n\pi x}{2} \quad \text{--- ②}$$

$$\text{To prove: i) } \frac{1}{1^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{8}$$

Put $x=0$ in ②, we get

$$f(0) = \frac{1}{4} + \sum_{n=1,3}^{\infty} \frac{2x}{n^2\pi^2} \quad \text{--- ③}$$

To find $f(0)$:

$x=0$ is an end of point of $(0, 2\pi)$

$$f(0) = 1 - 0 = 1$$

$$f(2\pi) = 0$$

$$\therefore f(0) \neq f(2\pi)$$

Here $x=0$ is a point of discontinuity,

$$\therefore f(0) = \frac{f(0) + f(2\pi)}{2}$$



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$$f(0) = \frac{1+0}{2} = \frac{1}{2}$$

Substitute $f(0) = \frac{1}{2}$ in (3), we get

$$\frac{1}{2} = \frac{1}{4} + \frac{2l}{\pi^2} \sum_{n=1,3}^{\infty} \frac{1}{n^2}$$

$$\frac{1}{2} - \frac{1}{4} = \frac{2l}{\pi^2} \sum_{n=1,3}^{\infty} \frac{1}{n^2}$$

$$\frac{1}{4} = \frac{2l}{\pi^2} \sum_{n=1,3}^{\infty} \frac{1}{n^2}$$

$$\frac{1}{4} \times \frac{\pi^2}{2l} = \sum_{n=1,3}^{\infty} \frac{1}{n^2}$$

$$\sum_{n=1,3}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{8}$$

$$\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \dots = \frac{\pi^2}{8}$$

ii) To deduce $1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$

Put $\alpha = \frac{1}{2}$ in (2), we get

$$f\left(\frac{1}{2}\right) = \frac{1}{4} + \sum_{n=1,3}^{\infty} \frac{2l}{n^2 \pi^2} \cos \frac{n\pi}{2} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \sin \frac{n\pi}{2}$$

$$f\left(\frac{1}{2}\right) = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \sin \frac{n\pi}{2} \quad \text{--- (4)}$$



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To find $f\left(\frac{1}{2}\right)$:

$x = \frac{1}{2}$ lies inside $(0, 2)$

Here $x = \frac{1}{2}$ is a point of continuity

$f\left(\frac{1}{2}\right) = \frac{1}{2}$ in (4), we get

$$\frac{1}{2} = \frac{1}{4} + \sum_{n=1}^{\infty} \frac{1}{n\pi} \sin \frac{n\pi}{2}$$

$$\frac{1}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} = \frac{1}{4} \cdot \pi$$

$$\sum_{n=1}^{\infty} \frac{1}{n} \sin \frac{n\pi}{2} = \frac{\pi}{4}$$

Ans: $1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$

~~Homework~~ Sum:

3) Find the Fourier series for $f(x) = 2x - x^2$,
 $0 < x < 3$

Solution:

Given interval is $(0, 3)$

General interval is $(0, 2\lambda)$

$$\therefore 2\lambda = 3 \Rightarrow \lambda = \frac{3}{2}$$



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The Fourier series for $f(x)$ in the interval $(0, 2\pi)$ is given by

$$\begin{aligned} f(x) &= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{n\pi x}{l} + \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{l} \\ &= \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2n\pi x}{3} + \sum_{n=1}^{\infty} b_n \sin \frac{2n\pi x}{3} \quad L \text{ (1)} \end{aligned}$$

$$\begin{aligned} a_0 &= \frac{1}{l} \int_0^{2l} f(x) dx \\ &= \frac{2}{3} \int_0^3 f(x) dx \\ &= \frac{2}{3} \int_0^3 (2x - x^2) dx \\ &= \frac{2}{3} \left[\frac{2x^2}{2} - \frac{x^3}{3} \right]_0^3 \\ &= \frac{2}{3} \left[x^2 - \frac{x^3}{3} \right]_0^3 \\ &= \frac{2}{3} \left[9 - \frac{27}{3} - (0) \right] \\ &= \frac{2}{3} (9 - 9) \end{aligned}$$

$$\boxed{a_0 = 0}$$

$$a_n = \frac{1}{l} \int_0^{2l} f(x) \cos \frac{n\pi x}{l} dx$$



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$$= \frac{2}{3} \int_0^3 f(x) \cos \frac{2n\pi x}{3} dx$$

$$= \frac{2}{3} \int_0^3 (2x - x^2) \cos \frac{2n\pi x}{3} dx$$

$$= \frac{2}{3} \left[(2x - x^2) \left(\frac{\sin \frac{2n\pi x}{3}}{\frac{2n\pi}{3}} \right) - (2 - 2x) \left(\frac{-\cos \frac{2n\pi x}{3}}{\left(\frac{2n\pi}{3} \right)^2} \right) + (-2) \left(\frac{-\sin \frac{2n\pi x}{3}}{\frac{2n\pi}{3}} \right) \right]_0^3$$

$$= \frac{2}{3} \left(\frac{3}{2n\pi} \right)^2 \left[(2 - 2x) \left(\cos \frac{2n\pi x}{3} \right) \right]_0^3$$

$$= \frac{3}{2n^2\pi^2} [(-4)(\cos 2n\pi - 2)(\cos 0)]$$

$$= \frac{3}{2n^2\pi^2} [-4 - 2]$$

$$\boxed{a_n = \frac{-9}{n^2\pi^2}}$$

$$b_n = \frac{1}{1} \int_0^{2\pi} f(x) \sin \frac{n\pi x}{1} dx$$

$$= \frac{2}{3} \int_0^3 (2x - x^2) \sin \frac{2n\pi x}{3} dx$$



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$$\begin{aligned}
&= \frac{2}{3} \left[(2x-x^2) \left(\frac{-\cos \frac{2n\pi x}{3}}{\left(\frac{2n\pi}{3}\right)} \right) - (2-x^2) \left(\frac{-\sin \frac{2n\pi x}{3}}{\left(\frac{2n\pi}{3}\right)^2} \right) \right. \\
&\quad \left. + (-2) \left(\frac{\cos \frac{2n\pi x}{3}}{\left(\frac{2n\pi}{3}\right)^3} \right) \right]_0^3 \\
&= \frac{2}{3} \left[\left(\frac{-3}{2n\pi} \right) (2x-x^2) \left(\cos \frac{2n\pi x}{3} \right) - 2 \left(\frac{3}{2n\pi} \right)^2 \left(\cos \frac{2n\pi x}{3} \right) \right]_0^3 \\
&= \frac{2}{3} \left\{ \left[\left(\frac{-3}{2n\pi} \right) (-3) \left(\cos 2n\pi \right) - 2 \left(\frac{27}{8n^3\pi^3} \right) \left(\cos 2n\pi \right) \right] \right. \\
&\quad \left. - \left[\left(\frac{-3}{2n\pi} \right) (0) \left(\cos 0 \right) - 2 \left(\frac{27}{8n^3\pi^3} \right) \left(\cos 0 \right) \right] \right\} \\
&= \frac{2}{3} \left\{ \left[\left(\frac{9}{2n\pi} \right) (1) - \left(\frac{27}{4n^3\pi^3} \right) (1) \right] - \left[0 - \frac{27}{4n^3\pi^3} (1) \right] \right\} \\
&= \frac{2}{3} \left[\frac{9}{2n\pi} - \frac{27}{4n^3\pi^3} + \frac{27}{4n^3\pi^3} \right]
\end{aligned}$$

$$b_n = \frac{3}{n\pi}$$

Substitute the values of a_0, a_n, b_n in ①

$$f(x) = \sum_{n=1}^{\infty} \frac{-9}{n^2\pi^2} \cos \frac{2n\pi x}{3} + \sum_{n=1}^{\infty} \frac{3}{n\pi} \sin \frac{2n\pi x}{3}$$



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Homework sums:

① Find the Fourier series for $f(x) = (1-x)^2$ to $0 < x < 2l$ hence find $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots$

$$\text{Ans: } \frac{l^2}{3} + \frac{4l^2}{\pi^2} \sum_{n=1}^{\infty} \frac{1}{n^2} \cos \frac{n\pi x}{l} ; \frac{\pi^2}{6}$$

② Find the Fourier series for $f(x) = (1-x)^2$, $0 < x < 2l$, find $\sum_{n=1}^{\infty} \frac{1}{n^2}$

$$\text{Ans: } f(x) = \frac{l^2}{3} + \sum_{n=1}^{\infty} \frac{4l^2}{n^2\pi^2} \cos \frac{n\pi x}{l}$$

③ Find the Fourier series expansion of $f(x) = 2lx - x^2$, $0 < x < 2l$

$$\text{Ans: } f(x) = \frac{2l^2}{3} + \sum_{n=1}^{\infty} \frac{-4l^2}{n^2\pi^2} \cos \frac{n\pi x}{l}$$