



SNS COLLEGE OF TECHNOLOGY

(An Autonomous Institution)

Approved by AICTE, New Delhi, Affiliated to Anna University, Chennai

Accredited by NAAC-UGC with 'A++' Grade (Cycle III) &

Accredited by NBA (B.E - CSE, EEE, ECE, Mech & B.Tech.IT)

COIMBATORE-641 035, TAMIL NADU



ODD AND EVEN FUNCTIONS:

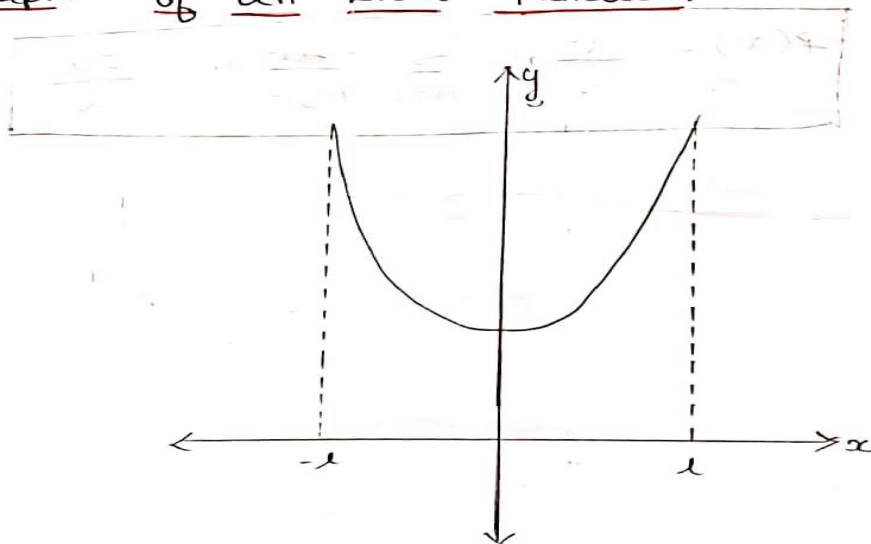
EVEN FUNCTION:

Let $f(x)$ be defined in $(-1, 1)$
If $f(-x) = f(x)$, then $f(x)$ is an even function.

Note:

1. The graph of even function is symmetrical about y-axis
2. $\int_{-1}^1 f(x) dx = 2 \int_0^1 f(x) dx$ if $f(x)$ is even.
3. Sum of two even functions is also an even function.
4. Product of two even functions is also an even function.

Graph of an Even Function:





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ODD FUNCTION:

Let $f(x)$ be defined in $(-a, a)$.
If $f(-x) = -f(x)$, then $f(x)$ is an odd function.

Note:

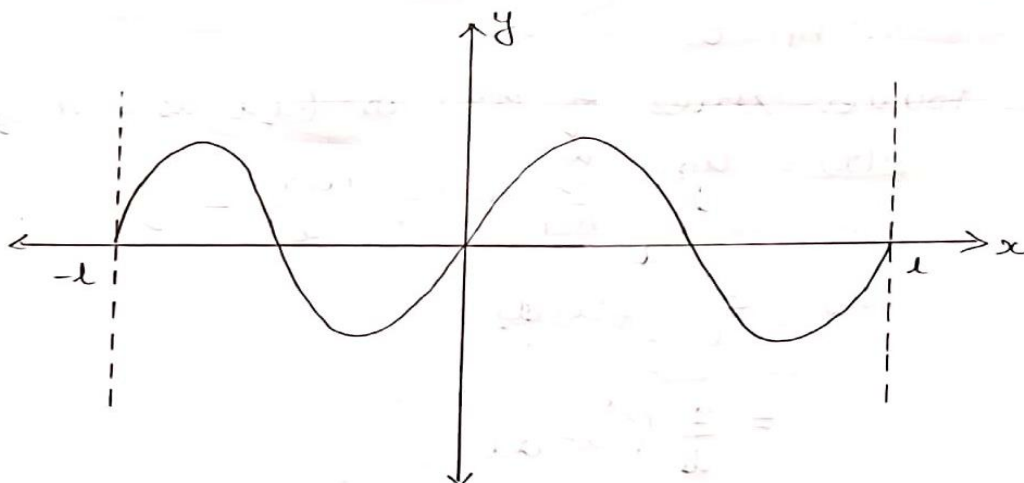
1. The graph of odd function is symmetric about origin.

2. $\int_{-a}^a f(x) dx = 0$ if $f(x)$ is odd

3. Sum of two odd function is also an odd function.

4. Product of two odd function is an even function.

Graph of an Odd function:





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③ Find the Fourier series for $f(x) = x$ in $(-1, 1)$ and hence deduce that

$$1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$$

Solution:

$f(x)$ is defined in $(-1, 1)$
In $(-1, 1)$, check whether $f(x)$ is odd/even.

$$f(x) = x$$

$$f(-x) = -x = -f(x)$$

$$\therefore f(-x) = -f(x)$$

$$\therefore \boxed{a_0 = a_n = 0}$$

The Fourier series is given by

For $f(x)$ in $(-1, 1)$

$$f(x) = \sum_{n=1}^{\infty} b_n \sin \frac{n\pi x}{1} \quad \text{--- (1)}$$

$$b_n = \frac{2}{1} \int_0^1 f(x) \sin \frac{n\pi x}{1} dx$$

$$= \frac{2}{1} \int_0^1 x \sin \frac{n\pi x}{1} dx$$

$$= \frac{2}{1} \left[x \left(\frac{-\cos \frac{n\pi x}{1}}{\left(\frac{n\pi}{1}\right)} \right) - (-1) \left(\frac{\sin \frac{n\pi x}{1}}{\left(\frac{n\pi}{1}\right)^2} \right) \right]_0^1$$

$$= \frac{-2}{1} \cdot \frac{1}{n\pi} \left[x \cos \frac{n\pi x}{1} \right]_0^1$$



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$$= \frac{-2}{n\pi} [l \cos n\pi - 0]$$

$$b_n = \frac{-2l}{n\pi} (-1)^n$$

Substitute the values of b_n in ①,
we get

$$f(x) = \sum_{n=1}^{\infty} \frac{-2l}{n\pi} (-1)^n \sin \frac{n\pi x}{l} \quad \text{--- (2)}$$

To deduce $1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}$.

Put $x = \frac{l}{2}$ in ② we get

$$f\left(\frac{l}{2}\right) = \sum_{n=1}^{\infty} \frac{-2l}{n\pi} (-1)^n \sin \frac{n\pi}{2} \quad \text{--- (3)}$$

To find $f\left(\frac{l}{2}\right)$:

$x = \frac{l}{2}$ lies inside $(-l, l)$.

Here $x = \frac{l}{2}$ is a point of continuity (

$$f\left(\frac{l}{2}\right) = \frac{l}{2}$$

$$\text{③} \Rightarrow \frac{l}{2} = \sum_{n=1}^{\infty} \frac{-2l}{n\pi} (-1)^n \sin \frac{n\pi}{2}$$

$$-\frac{2l}{\pi} \sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} = \frac{l}{2}$$



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$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} = \frac{1}{2} \left(\frac{\pi}{-2i} \right)$$

$$\sum_{n=1}^{\infty} \frac{(-1)^n}{n} \sin \frac{n\pi}{2} = -\frac{\pi}{4}$$

$$\frac{(-1)}{1} (1) + 0 + \frac{(-1)}{3} (-1) + \dots = -\frac{\pi}{4}$$

Ans : $\boxed{1 - \frac{1}{3} + \frac{1}{5} - \dots = \frac{\pi}{4}}$

Homework sums:

① Find the fourier series for $f(x) = x^2$ in $(-\pi, \pi)$ and hence deduce

$$i) \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{6} \quad ii) \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots = \frac{\pi^2}{12}$$

Ans: $\boxed{f(x) = \frac{\pi^2}{3} + \sum_{n=1}^{\infty} \frac{4}{n^2} (-1)^n \cos nx}$

② Find the Fourier series for

$$f(x) = \begin{cases} 1 + \frac{x}{\pi} & ; -\pi < x < 0 \\ 1 - \frac{x}{\pi} & ; 0 < x < \pi \end{cases}$$

and hence

deduce $\frac{1}{1^2} + \frac{1}{3^2} + \dots = \frac{\pi^2}{8}$

Ans: $\boxed{f(x) = \frac{1}{2} + \sum_{n=1,3,5}^{\infty} \frac{4}{n^2 \pi^2} \cos nx}$