



PROBLEMS USING PERSEVAL'S IDENTITY
IN FOURIER SINE TRANSFORM &
FOURIER COSINE TRANSFORM.

$$\int_0^{\infty} F_s(s) G_s(s) ds = \int_0^{\infty} f(x) g(x) dx$$

where, $F_s(s) = F_s[f(x)]$

$$G_s(s) = F_s[g(x)]$$

$$\int_0^{\infty} F_c(s) G_c(s) ds = \int_0^{\infty} f(x) g(x) dx$$

where, $F_c(s) = F_c[f(x)]$

$$G_c(s) = F_c[g(x)]$$

① Evaluate $\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)}$; $a, b > 0$

Solution:

Since the numerator in the integrant is dx , we use Fourier cosine transform. Since denominator $(x^2+a^2)(x^2+b^2)$ we take $f(x) = e^{-ax}$; $g(x) = e^{-bx}$

$$F_c(s) = F_c[f(x)]$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \cos sx \, dx$$

$$F_c(s) = \sqrt{\frac{2}{\pi}} \frac{a}{a^2 + s^2}$$

Similarly

$$G_c(s) = \sqrt{\frac{2}{\pi}} \frac{b}{b^2 + s^2}$$

Parseval's Identity in Fourier cosine transform

$$\int_0^{\infty} F_c(s) G_c(s) \, ds = \int_0^{\infty} f(x) g(x) \, dx$$

$$\int_0^{\infty} \sqrt{\frac{2}{\pi}} \frac{a}{a^2 + s^2} \sqrt{\frac{2}{\pi}} \frac{b}{s^2 + b^2} \, ds = \int_0^{\infty} e^{-ax} e^{-bx} \, dx$$

$$\frac{2}{\pi} ab \int_0^{\infty} \left(\frac{1}{a^2 + s^2} \right) \left(\frac{1}{s^2 + b^2} \right) \, ds = \int_0^{\infty} e^{-(a+b)x} \, dx$$

$$\frac{2}{\pi} ab \int_0^{\infty} \frac{ds}{(s^2 + a^2)(s^2 + b^2)} = \left[\frac{e^{-(a+b)x}}{-(a+b)} \right]_0^{\infty}$$

$$= 0 - \frac{1}{-(a+b)}$$

$$\frac{2}{\pi} ab \int_0^{\infty} \frac{ds}{(s^2 + a^2)(s^2 + b^2)} = \frac{1}{a+b}$$

$$\int_0^{\infty} \frac{ds}{(s^2 + a^2)(s^2 + b^2)} = \frac{\pi}{2} \frac{1}{ab(a+b)}$$



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Put $s=x$; $ds=dx$

$$\int_0^{\infty} \frac{dx}{(x^2+a^2)(x^2+b^2)} = \frac{\pi}{2ab(a+b)}$$

② Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)(x^2+b^2)}$; $a>0$; $b>0$

Solution:

Since the numerator of the integrant is $x^2 dx$, so we use Fourier sine transform.

Denominator is $(x^2+a^2)(x^2+b^2)$

Take $f(x) = e^{-ax}$; $g(x) = e^{-bx}$

$$F_s(s) = F_s[f(x)]$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \sin sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \frac{s}{a^2+s^2}$$

Similarly

$$G_s(s) = \sqrt{\frac{2}{\pi}} \frac{s}{b^2+s^2}$$



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Parseval's Identity in Fourier sine transforms

$$\int_0^{\infty} F_s(s) G_s(s) ds = \int_0^{\infty} f(x) g(x) dx$$

$$\int_0^{\infty} \sqrt{\frac{2}{\pi}} \frac{s}{a^2 + s^2} \sqrt{\frac{2}{\pi}} \frac{s}{b^2 + s^2} ds = \int_0^{\infty} e^{-ax} b e^{-bx} dx$$

$$\frac{2}{\pi} \int_0^{\infty} \frac{s}{a^2 + s^2} \frac{s}{s^2 + b^2} ds = \int_0^{\infty} e^{-(a+b)x} dx$$

$$= \left[\frac{e^{-(a+b)x}}{-(a+b)} \right]_0^{\infty} = \frac{1}{(a+b)}$$

$$\int_0^{\infty} \frac{s^2}{(s^2 + a^2)(s^2 + b^2)} ds = \frac{\pi}{2(a+b)}$$

Put $s = x$; $ds = dx$

③ Use Parseval's Identity for Fourier cosine and sine transform for e^{-ax} , evaluate

$$(i) \int_0^{\infty} \frac{dx}{(x^2 + a^2)^2}$$

$$(ii) \int_0^{\infty} \frac{x^2 dx}{(x^2 + a^2)^2}$$



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Solution:

is To evaluate $\int_0^{\infty} \frac{dx}{(x^2+a^2)^2}$

Since numerator of the integrand is dx ,
use cosine transform.

Denominator is

$$(x^2 + a^2)^2 = (x^2 + a^2)(x^2 + a^2)$$

$$\text{Here } f(x) = e^{-ax} ; g(x) = e^{-ax}$$

$$F_c(s) = F_c[f(x)]$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \cos sx \, dx$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \cos sx \, dx$$

$$F_c(s) = \sqrt{\frac{2}{\pi}} \frac{a}{a^2 + s^2}$$

$$G_c(s) = \sqrt{\frac{2}{\pi}} \frac{a}{a^2 + s^2}$$

Parseval's Identity in Fourier cosine transform is

$$\int_0^{\infty} F_c(s) G_c(s) \, ds = \int_0^{\infty} f(x) g(x) \, dx$$



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$$\int_0^{\infty} \sqrt{\frac{2}{\pi}} \frac{a}{a^2+s^2} \sqrt{\frac{2}{\pi}} \frac{a}{a^2+b^2} ds = \int_0^{\infty} e^{-2ax} dx$$

$$\frac{2a^2}{\pi} \int_0^{\infty} \frac{ds}{(a^2+s^2)^2} = \int_0^{\infty} e^{-2ax} dx$$

$$= \left[\frac{e^{-2ax}}{-2a} \right]_0^{\infty}$$

$$\frac{2a^2}{\pi} \int_0^{\infty} \frac{ds}{(a^2+s^2)^2} = \frac{1}{2a}$$

$$\int_0^{\infty} \frac{ds}{(s^2+a^2)^2} = \frac{\pi}{4a^3}$$

Put $s=x$; $ds = dx$

ii) To evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)^2}$

Since the numerator of the integrant is $x^2 dx$, we use sine transform

Denominator is $(x^2+a^2)^2 = (x^2+a^2)(x^2+a^2)$

Take $f(x) = e^{-ax}$; $g(x) = e^{-ax}$

$$F_s(s) = F_s[f(x)]$$

$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx dx$$



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$$= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \sin sx \, dx$$

$$F_S(s) = \sqrt{\frac{2}{\pi}} \frac{s}{a^2 + s^2}$$

similarly,

$$G_L(s) = \sqrt{\frac{2}{\pi}} \frac{s}{a^2 + s^2}$$

Parseval's Identity in Fourier sine transforms

$$\int_0^{\infty} F_S(s) G_L(s) \, ds = \int_0^{\infty} f(x) g(x) \, dx$$

$$\int_0^{\infty} \sqrt{\frac{2}{\pi}} \frac{s}{a^2 + s^2} \sqrt{\frac{2}{\pi}} \frac{s}{s^2 + a^2} \, ds = \int_0^{\infty} e^{-ax} e^{-ax} \, dx$$

$$\frac{2}{\pi} \int_0^{\infty} \frac{s^2}{(a^2 + s^2)(a^2 + s^2)} \, ds = \int_0^{\infty} e^{-2ax} \, dx$$

$$= \left[\frac{e^{-2ax}}{-2a} \right]_0^{\infty}$$

$$= 0 - \left(\frac{1}{-2a} \right)$$

$$\frac{2}{\pi} \int_0^{\infty} \frac{s^2}{(s^2 + a^2)^2} \, ds = \frac{1}{2a}$$



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$$\int_0^{\infty} \frac{s^2}{(s^2+a^2)^2} ds = \frac{\pi}{4a}$$

Put $s=x \Rightarrow ds=dx$

$$\int_0^{\infty} \frac{x^2 dx}{(x^2+a^2)^2} = \frac{\pi}{4a}$$